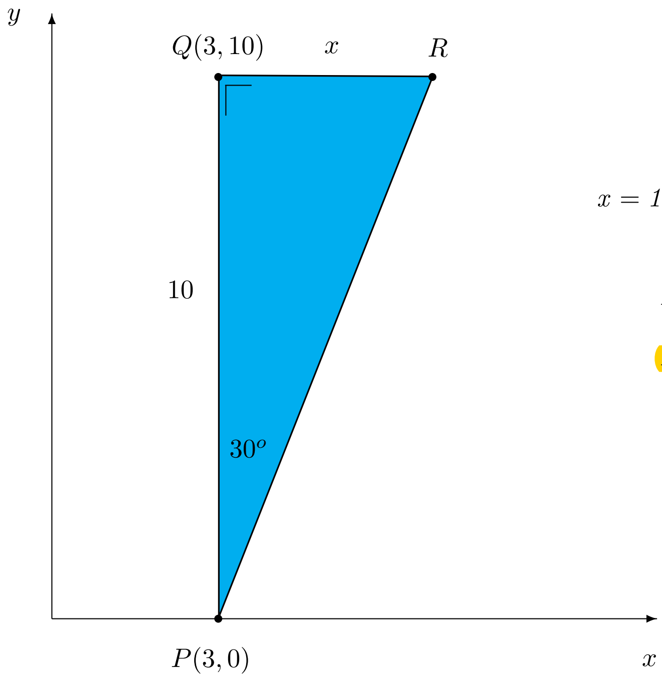
**1.** In the following coordinate diagrams in $\mathbb{R}^2$ the triangle PQR is a right-angled triangle. Using *basic trigonometry*, determine the coordinates of the point $R$.

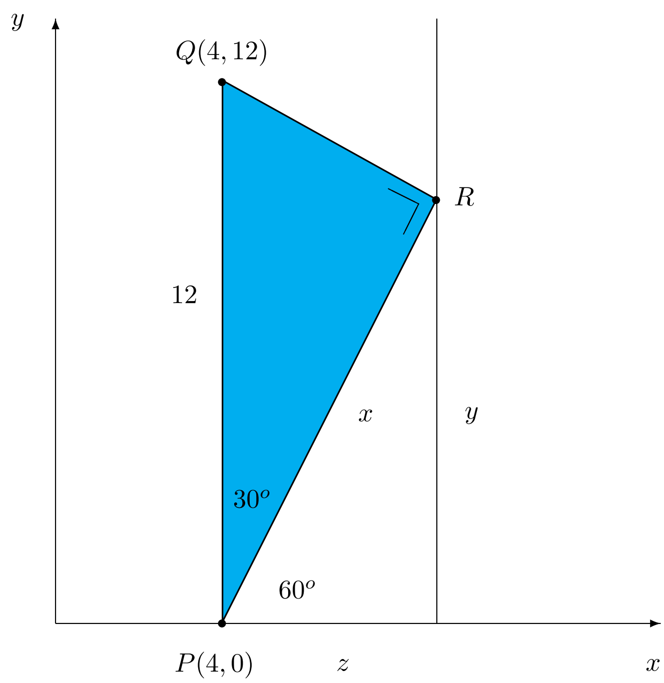



$$\tan 30^o = x/10$$

$$x = 10 \tan 30^o = 10\sqrt{3}/3$$

$$R = (3 + 10\sqrt{3}/3, 10)$$

$$R = (8 \cdot 77, 10)$$




$$\cos 30^o = x/12$$

$$x = 12 \cos 30^o = 6\sqrt{3}$$

$$\sin 60^o = y/6\sqrt{3}$$

$$y = 6\sqrt{3} \sin 60^o = 9$$

$$\cos 60^o = z/6\sqrt{3}$$

$$z = 6\sqrt{3} \cos 60^o = 3\sqrt{3}$$

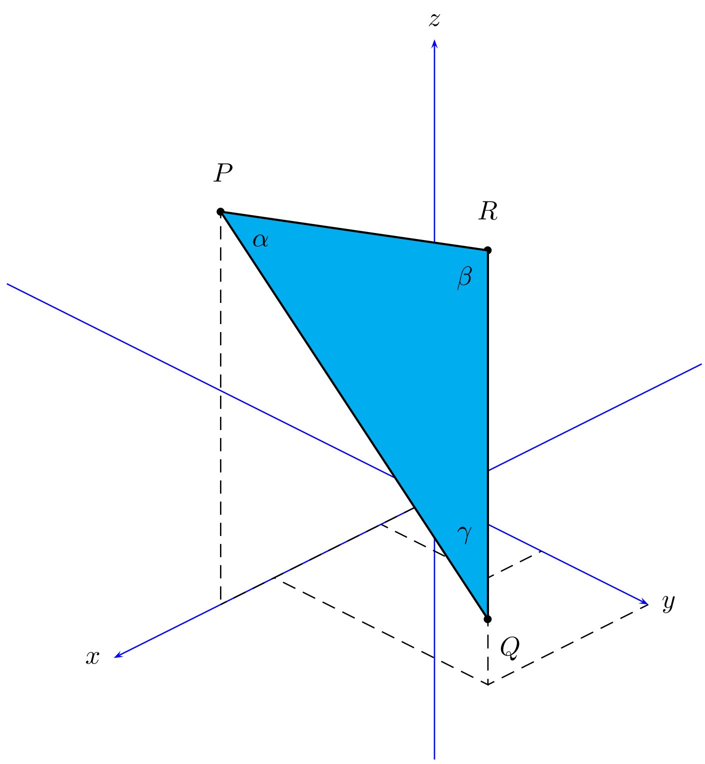
$$R = (4 + 3\sqrt{3}, 9)$$

$$R = (9 \cdot 12, 9)$$

**30 marks**

**2.** Consider the following points in $\mathbb{R}^3$.

$$P(1,2,-3) \quad , \quad Q(3,2,1) \quad , \quad R(4,0,-2)$$




i Evaluate $P - 2Q + 3R$.

ii Determine $||2P - 4Q||$.

iii Evaluate $P.Q$, i.e., the *scalar product* of P with Q.

iv Evaluate $P \times Q$.

v Show that the vectors defining this triangle $\triangle PQR$ are *coplanar*.

vi Determine all internal angles $\alpha, \beta, \gamma$ of the triangle $\triangle PQR$.

vii Determine the area of the triangle $\triangle PQR$.

**70 marks**

*i*

$$P - 2Q + 3R = (1,2,-3) - 2(3,2,1) + 3(4,0,-2) = (7,-2,-11)$$

*ii*

$$\begin{aligned} 2P - 4Q &= 2(1,2,-3) - 4(3,2,1) = (-10,-4,-10) \\ ||2P - 4Q|| &= \sqrt{100+16+100} = \sqrt{216} \end{aligned}$$

*iii*

$$P.Q = 1.3 + 2.2 + (-3).1 = 4$$

*iv*

$$P \times Q = det \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -3 \\ 3 & 2 & 1 \end{vmatrix} = (8,-10,-4)$$

*v*

$$\begin{aligned} \vec{PQ} &= \vec{Q} - \vec{P} = (3,2,1) - (1,2,-3) = (2,0,4) \\ \vec{PR} &= \vec{R} - \vec{P} = (4,0,-2) - (1,2,-3) = (3,-2,1) \\ \vec{RQ} &= \vec{Q} - \vec{R} = (3,2,1) - (4,0,-2) = (-1,2,3) \end{aligned}$$

$$\vec{PQ} \times \vec{PR} = det \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 4 \\ 3 & -2 & 1 \end{vmatrix} = (8,10,-4)$$

$$\vec{RQ}.(\vec{PQ} \times \vec{PR}) = -1(8) + 2(10) + 3(-4) = -8 + 20 - 12 = 0$$

*The vectors defining this triangle $\triangle PQR$ are coplanar.*

*vi* *To determine the angle $\alpha$*

$$\cos\alpha = \frac{\vec{PQ}.\vec{PR}}{\|\vec{PQ}\|\|\vec{PR}\|} = \frac{10}{\sqrt{20}.\sqrt{14}} = 0 \cdot 5976 \quad , \quad \alpha = 53 \cdot 3^o$$

*To determine the angle $\beta$*

$$\cos\beta = \frac{\vec{RP}.\vec{RQ}}{\|\vec{RP}\|\|\vec{RQ}\|} = \frac{4}{\sqrt{14}.\sqrt{14}} = 0 \cdot 2857 \quad , \quad \beta = 73 \cdot 4^o$$

*Finally $\alpha + \beta + \gamma = 180^o$, hence $\gamma = 53 \cdot 3^o$.*

vii *Hence the area of the triangle* $\triangle PQR$ *is*

$$\frac{1}{2}\|\vec{PQ}\times\vec{PR}\|$$

*where* $\vec{PQ}\times\vec{PR} = (8, 10, -4)$. *Now*

$$\frac{1}{2}\|\vec{PQ}\times\vec{PR}\| = \frac{1}{2}\sqrt{64+100+16} = \frac{1}{2}\sqrt{180}$$

*The area of the triangle* $\triangle PQR$ *is* $3\sqrt{5}$ *sq. units.*