1. This coordinate diagrams in $\mathbb{R}^2$ shows a right-angled triangle PQR.

Using *basic trigonometry*, determine the coordinate R.

$$\cos 30^o = x/8$$

$$x = 8\cos 30^o = 4\sqrt{3}$$

$$\sin 60^o = y/4\sqrt{3}$$

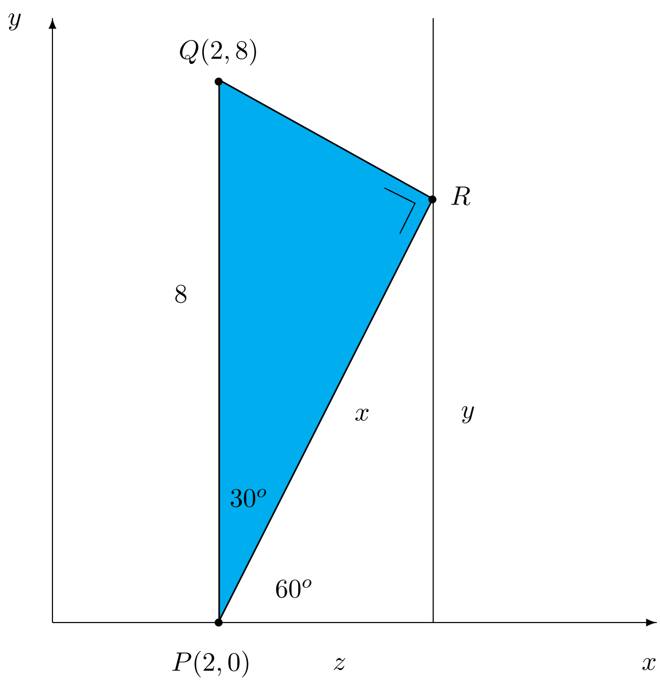
$$y = 4\sqrt{3}\sin 60^o = 6$$

$$\cos 60^o = z/4\sqrt{3}$$

$$z = 4\sqrt{3}\cos 60^o = 2\sqrt{3}$$

$$R = (2 + 2\sqrt{3}, 6)$$

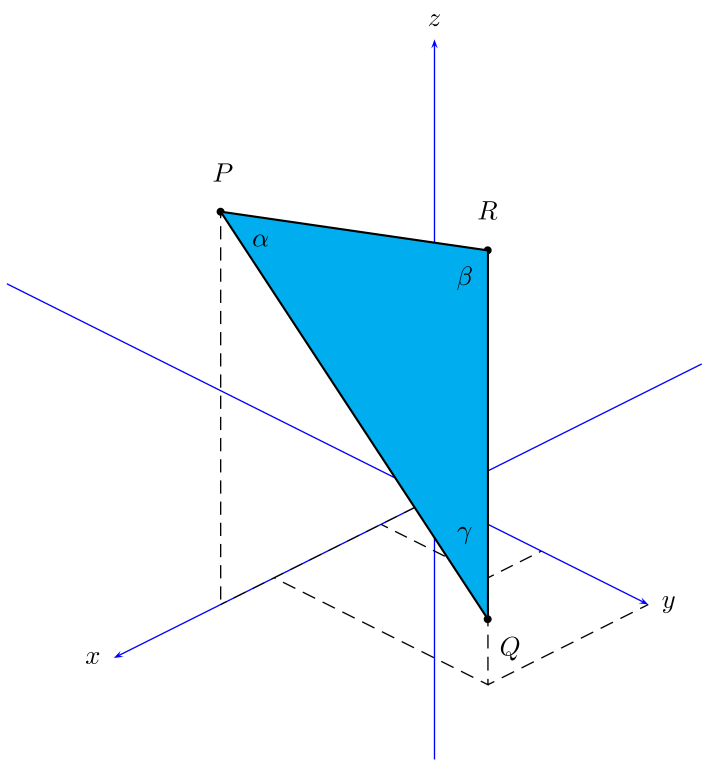
$$R = (5\cdot 46, 6)$$



20 marks

2. Consider the following points in $\mathbb{R}^3$.

$$P(3, 2, -1) \quad , \quad Q(6, 6, 0) \quad , \quad R(4, 0, 5)$$



i Evaluate $P - 2Q + 3R$.

ii Determine $||2P - 4Q||$.

iii Evaluate $P.Q$, i.e., the *scalar product* of P with Q.

iv Evaluate $P \times Q$.

v Show that the vectors defining this triangle $\triangle PQR$ are *coplanar*.

vi Determine all internal angles α, β, γ of the triangle $\triangle PQR$.

vii Determine the area of the triangle $\triangle PQR$.

i

$$P - 2Q + 3R = (3, 2, -1) - 2(6, 6, 0) + 3(4, 0, 5) = (3, -10, 14)$$

ii

$$\begin{aligned} 2P - 4Q &= 2(3, 2, -1) - 4(6, 6, 0) = (-18, -20, -2) \\ ||2P - 4Q|| &= \sqrt{324 + 400 + 4} = \sqrt{728} \end{aligned}$$

iii

$$P.Q = 3.6 + 2.6 + (-1).0 = 30$$

iv

$$P \times Q = det \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 2 & -1 \\ 6 & 6 & 0 \end{vmatrix} = (6, -6, 6)$$

v

$$\begin{aligned} \vec{PQ} &= \vec{Q} - \vec{P} = (6, 6, 0) - (3, 2, -1) = (3, 4, 1) \\ \vec{PR} &= \vec{R} - \vec{P} = (4, 0, 5) - (3, 2, -1) = (1, -2, 6) \\ \vec{RQ} &= \vec{Q} - \vec{R} = (6, 6, 0) - (4, 0, 5) = (2, 6, -5) \end{aligned}$$

$$\vec{PQ} \times \vec{PR} = det \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 4 & 1 \\ 1 & -2 & 6 \end{vmatrix} = (26, -17, -10)$$

$$\vec{RQ}.(\vec{PQ} \times \vec{PR}) = 2(26) + 6(-17) - 5(-10) = 52 - 102 + 50 = 0$$

The vectors defining this triangle $\triangle PQR$ are coplanar.

vi To determine the angle α

$$\cos\alpha = \frac{\vec{PQ}.\vec{PR}}{\|\vec{PQ}\|\|\vec{PR}\|} = \frac{1}{\sqrt{26}.\sqrt{41}} = 0 \cdot 0306 \quad , \quad \alpha = 88 \cdot 2^o$$

To determine the angle β

$$\cos\beta = \frac{\vec{RP}.\vec{RQ}}{\|\vec{RP}\|\|\vec{RQ}\|} = \frac{40}{\sqrt{41}.\sqrt{65}} = 0 \cdot 7748 \quad , \quad \beta = 39 \cdot 2^o$$

Finally $\alpha + \beta + \gamma = 180^o$, hence $\gamma = 52 \cdot 6^o$.

vii Hence the area of the triangle $\triangle PQR$ is

$$\frac{1}{2}\|\vec{PQ} \times \vec{PR}\|$$

where $\vec{PQ} \times \vec{PR} = (26, -17, -10)$. Now

$$\frac{1}{2}\|\vec{PQ} \times \vec{PR}\| = \frac{1}{2}\sqrt{676 + 289 + 100} = \frac{1}{2}\sqrt{1354}$$

The area of the triangle $\triangle PQR$ is $18 \cdot 4$ *sq. units*.

80 marks