INSTRUCTIONS

Full marks will be awarded for the correct solutions to BOTH QUESTIONS. This paper will be marked out of a TOTAL MAXIMUM MARK OF 100. Credit will be given for clearly presented solutions.

BSc (Hons) in Computer Games Development

YEAR 1

CLASS TEST 1 (SOLUTIONS)

1. *i*

$$\begin{aligned} \vec{u} - 4\vec{v} + 2\vec{w} &= (2,1,3) - 4(4,-1,0) + 2(2,0,1) \\ &= (2,1,3) - (16,-4,0) + (4,0,2) \\ &= (-10,5,5) \end{aligned}$$

ii Firstly,

$$\begin{aligned} 7\vec{u} - 2\vec{v} &= 7(2,1,3) - 2(4,-1,0) \\ &= (14,7,21) - (8,-2,0) \\ &= (6,9,21) \end{aligned}$$

Now

$$\begin{aligned} ||7\vec{u} - 2\vec{v}|| &= \sqrt{6^2 + 9^2 + 21^2} \\ &= \sqrt{558} \end{aligned}$$

iii

$$\begin{aligned} \vec{u}.\vec{v} &= (2,1,3).(4,-1,0) \\ &= 2.(4) + 1.(-1) + 3.(0) \\ &= 8 - 1 + 0 \\ &= 7 \end{aligned}$$

iv Firstly

$$\begin{aligned} ||\vec{u}|| &= \sqrt{2^2+1^2+3^2} = \sqrt{14} \\ ||\vec{v}|| &= \sqrt{4^2+(-1)^2+0^2} = \sqrt{17} \end{aligned}$$

Recall

$$\cos\theta = \frac{\vec{u}.\vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \frac{7}{\sqrt{14}\sqrt{17}}$$

Hence

$$\theta = \cos^{-1}(0\cdot 4537) = 63\cdot 02^o$$

v Recall

$$proj_{\vec{v}}(\vec{u}) = \left(\frac{\vec{u}.\vec{v}}{\|\vec{v}\|}\right)\frac{\vec{v}}{\|\vec{v}\|} = \left(\frac{7}{\sqrt{17}}\right).\frac{(4,-1,0)}{\sqrt{17}} = \frac{7}{17}(4,-1,0)$$

vi Firstly,

$$\vec{u}\times\vec{v} = det\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 3 \\ 4 & -1 & 0 \end{vmatrix} = (3,12,-6)$$

Area of parallelogram $= ||\vec{u}\times\vec{v}||$.

$$\begin{aligned} ||\vec{u}\times\vec{v}|| &= \sqrt{3^2+12^2+(-6)^2} \\ &= \sqrt{189} \end{aligned}$$

vii Firstly,

$$\vec{v}\times\vec{w} = det\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -1 & 0 \\ 2 & 0 & 1 \end{vmatrix} = (-1,-4,2)$$

Finally,

$$\begin{aligned} \vec{u}.(\vec{v}\times\vec{w}) &= (2,1,3).(-1,-4,2) \\ &= 2.(-1)+1.(-4)+3.(2) \\ &= -2-4+6 \\ &= 0 \end{aligned}$$

Therefore, $\vec{u},\vec{v}$ *and* $\vec{w}$ *are coplanar.*

2. *i*

$$\begin{aligned}\vec{AB} &= \vec{B}-\vec{A} = (2,1,-1)-(-1,0,2) = (3,1,-3)\\ \vec{AC} &= \vec{C}-\vec{A} = (1,-2,2)-(-1,0,2) = (2,-2,0)\\ \vec{BC} &= \vec{C}-\vec{B} = (1,-2,2)-(2,1,-1) = (-1,-3,3)\end{aligned}$$

Now

$$\vec{AC}\times\vec{BC} = det\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2 & -2 & 0\\ -1 & -3 & 3\end{vmatrix} = (-6,-6,-8)$$

Finally

$$\vec{AB}.(\vec{AC}\times\vec{BC}) = 3(-6)+1(-6)-3(-8) = -18-6+24 = 0$$

Therefore the vectors defining this triangle $\triangle ABC$ *are coplanar.*

ii *To determine the angle* α *we require vectors* $\vec{AB}$ *and* $\vec{AC}$*. Now*

$$\begin{aligned}\vec{AB} &= (3,1,-3)\\ \vec{AC} &= (2,-2,0)\end{aligned}$$

Now

$$\cos\alpha = \frac{\vec{AB}.\vec{AC}}{\|\vec{AB}\|\|\vec{AC}\|} = \frac{4}{\sqrt{19}\ .\ \sqrt{8}} = 0\cdot 3244$$

$$\alpha = cos^{-1}(0\cdot 3244)$$

$$\alpha = 71.1^o$$

To determine the angle β *we require vectors* $\vec{BA}$ *and* $\vec{BC}$*. Now*

$$\begin{aligned}\vec{BA} &= (-3,-1,3)\\ \vec{BC} &= (-1,-3,3)\end{aligned}$$

Now

$$\cos\beta = \frac{\vec{BA}.\vec{BC}}{\|\vec{BA}\|\|\vec{BC}\|} = \frac{15}{\sqrt{19}\ .\ \sqrt{19}} = 0\cdot 7894$$

$$\beta = cos^{-1}(0\cdot 7894)$$

$$\beta = 37.9^o$$

Finally

$$\alpha + \beta + \gamma = 180^o$$

Hence $\gamma = 71^o$.

iii *To determine the area of the triangle* $\triangle ABC$ *note the following theorem*

Suppose that $\vec{u}, \vec{v} \in \mathbb{R}^3$. *Then the parallelogram with* $\vec{u}$ *and* $\vec{v}$ *as two of its sides has area* $\|\vec{u} \times \vec{v}\|$.

Hence the area of the triangle $\triangle ABC$ *is*

$$\frac{1}{2}\|\vec{AC} \times \vec{BC}\|$$

Now

$$\vec{AC} \times \vec{BC} = det \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -2 & 0 \\ -1 & -3 & 3 \end{vmatrix} = (-6, -6, -8)$$

Finally

$$\frac{1}{2}\|\vec{AC} \times \vec{BC}\| = \frac{1}{2}\sqrt{(-6)^2 + (-6)^2 + (-8)^2} = \frac{1}{2}\sqrt{136}$$

The area of the triangle is

$$\sqrt{34} \ sq. \ units$$