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INSTITUTE of TECHNOLOGY CARLOW
At the Heart of South Leinster

INSTRUCTIONS

Full marks will be awarded for the correct solutions to BOTH QUESTIONS. This paper will be marked out of a TOTAL MAXIMUM MARK OF 100. Credit will be given for clearly presented solutions.

BSc (Hons) in Computer Games Development

YEAR 1

CLASS TEST 1

1. Consider the following vectors $\vec{u} = (2, 1, 3)$, $\vec{v} = (4, -1, 0)$ and $\vec{w} = (2, 0, 1)$ in $\mathbb{R}^3$.

i Evaluate $\vec{u} - 4\vec{v} + 2\vec{w}$

ii Determine $||7\vec{u} - 2\vec{v}||$

iii Evaluate $\vec{u}.\vec{v}$

iv Determine the angle θ between $\vec{u}$ and $\vec{v}$.

v Determine the *vector projection* of $\vec{u}$ along $\vec{v}$, i.e., $proj_{\vec{v}}(\vec{u})$

vi Evaluate $\vec{u} \times \vec{v}$.

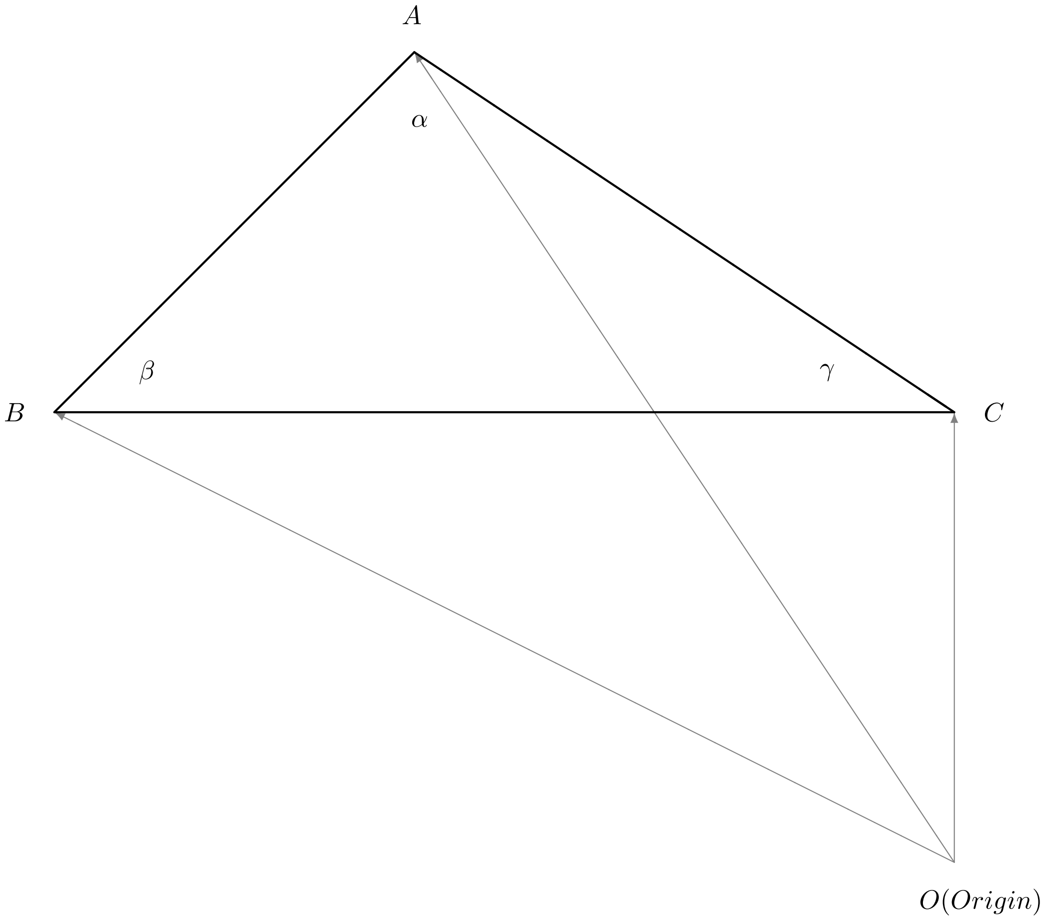
vii Determine the area of the *parallelogram* that is defined by $\vec{u}$ and $\vec{v}$.

viii Determine $\vec{u}.(\vec{v} \times \vec{w})$. What comment can you make about the vectors $\vec{u}, \vec{v}$ and $\vec{w}$?

50 marks

2. Consider the following points in $\mathbb{R}^3$.

$$A(-1,0,2) \quad , \quad B(2,1,-1) \quad , \quad C(1,-2,2)$$



i Show that the vectors defining this triangle $\triangle ABC$ are *coplanar*.

ii Determine all internal angles α, β, γ of the triangle $\triangle ABC$.

iii Determine the area of the triangle $\triangle ABC$.

50 marks

USEFUL INFORMATION

VECTORS IN $\mathbb{R}^3$:

For any vector $\vec{u} = (u_1, u_2, u_3)$ in $\mathbb{R}^3$, we define the magnitude of $\vec{u}$ to be the non-negative real number

$$\|\vec{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$$

———————o———————

Suppose that $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$ are vectors in $\mathbb{R}^3$ and that $\theta \in [0, \pi]$ represents the angle between them. We define the scalar product $\vec{u}.\vec{v}$ of $\vec{u}$ and $\vec{v}$ by $\vec{u}.\vec{v} = \|\vec{u}\|\|\vec{v}\| \cos\theta$. Hence

$$\cos\theta = \frac{\vec{u}.\vec{v}}{\|\vec{u}\|\|\vec{v}\|}$$

Note that the scalar product may also be defined as

$$\vec{u}.\vec{v} = u_1v_1 + u_2v_2 + u_3v_3$$

———————o———————

The *vector projection* of $\vec{u}$ along $\vec{v}$, denoted by $proj_{\vec{v}}(\vec{u})$, is the vector defined by

$$proj_{\vec{v}}(\vec{u}) = \left(\frac{\vec{u}.\vec{v}}{\|\vec{v}\|}\right)\frac{\vec{v}}{\|\vec{v}\|}$$

———————o———————

Theorem 1

Suppose that $\vec{u}, \vec{v} \in \mathbb{R}^3$. Then the parallelogram with $\vec{u}$ and $\vec{v}$ as two of its sides has area $\|\vec{u} \times \vec{v}\|$.