1. Let

$$A.B = \frac{1}{2}\begin{pmatrix} -4 & 8 & 2 \\ -3 & 5 & 3 \\ -2 & 4 & 0 \end{pmatrix}\begin{pmatrix} 6 & -4 & -7 \\ 3 & -2 & -3 \\ 1 & 0 & -2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

A is *invertible*.

It has an *inverse* given as B.

20 marks

2.

$$A = \begin{pmatrix} -5 & 1 & 4 \\ -1 & 1 & 1 \\ -4 & 1 & 3 \end{pmatrix}$$

We form the following matrix

$$\left(\begin{array}{ccc|ccc} -5 & 1 & 4 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ -4 & 1 & 3 & 0 & 0 & 1 \end{array}\right)$$

We now apply row operations to this 3×6 matrix $[A|I_n]$ to obtain the row equivalent *row-reduced echelon matrix* $[I_n|A^{-1}]$

$$\frac{R_1 \leftrightarrow R_2}{(-1)R_1}\left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & -1 & 0 \\ -5 & 1 & 4 & 1 & 0 & 0 \\ -4 & 1 & 3 & 0 & 0 & 1 \end{array}\right)$$

$$\frac{R_2 + 5R_1}{R_3 + 4R_1}\left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & -1 & 0 \\ 0 & -4 & -1 & 1 & -5 & 0 \\ 0 & -3 & -1 & 0 & -4 & 1 \end{array}\right)$$

$$\frac{R_2 - R_3}{}\left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 & -1 & -1 \\ 0 & -3 & -1 & 0 & -4 & 1 \end{array}\right)$$

$$\frac{R_3 - 3R_2}{}\left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 & -1 & -1 \\ 0 & 0 & -1 & -3 & -1 & 4 \end{array}\right)$$

$$\frac{(-1)R_2}{(-1)R_3}\left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 3 & 1 & -4 \end{array}\right)$$

$$\xrightarrow{R_1 + R_3} \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 3 & 0 & -4 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 3 & 1 & -4 \end{array}\right)$$

$$\xrightarrow{R_1 + R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & -3 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 3 & 1 & -4 \end{array}\right)$$

Therefore, we have

$$A^{-1} = \begin{pmatrix} 2 & 1 & -3 \\ -1 & 1 & 1 \\ 3 & 1 & -4 \end{pmatrix}$$

For a system of n linear equations in n unknowns represented as $Ax = B$, we can now use the inverse A^{-1} to solve the system using matrix multiplication as follows.

$$\begin{aligned} A^{-1}Ax &= A^{-1}B \\ \Rightarrow \quad Ix &= A^{-1}B \\ \therefore \quad x &= A^{-1}B \end{aligned}$$

Hence

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & 1 & -3 \\ -1 & 1 & 1 \\ 3 & 1 & -4 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$$

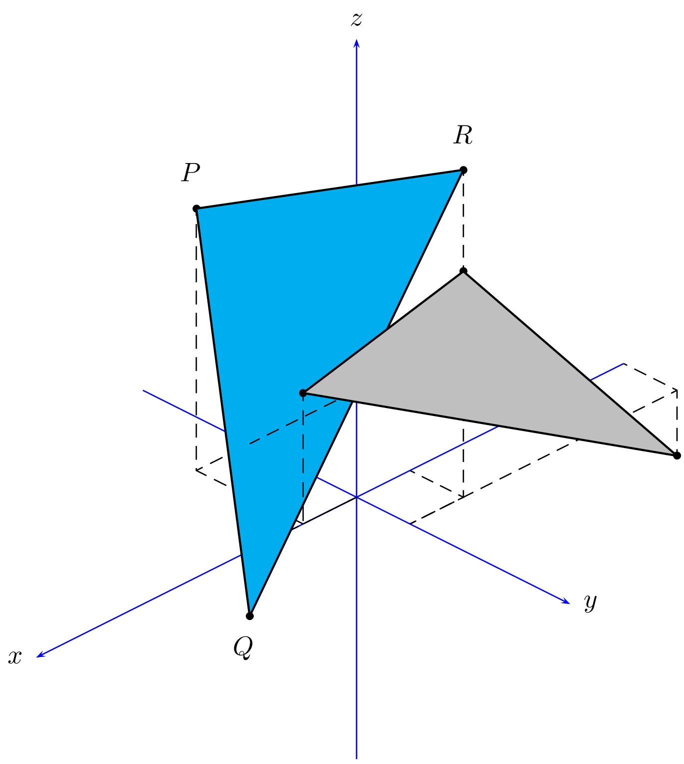
Finally, our solution is

$$x = -1,\ y = 3,\ z = -2$$

50 marks

3. Consider the following points in $\mathbb{R}^3$.

$$P(7,2,3) \quad , \quad Q(3,-2,1) \quad , \quad R(4,0,9)$$



To rotate the triangle PQR using A.

$$P' = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 7 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -7 \\ 2 \\ -3 \end{pmatrix}$$

$$Q' = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \\ -1 \end{pmatrix}$$

$$R' = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \\ 9 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ -9 \end{pmatrix}$$

30 marks