INSTRUCTIONS

Full marks will be awarded for the correct solutions to **ALL QUESTIONS**. This paper will be marked out of a TOTAL MAXIMUM MARK OF 100. Credit will be given for clearly presented solutions.

BSc (Hons) in Computer Games Development

YEAR 1

CLASS TEST 2 (SOLUTIONS)

1. Let

$$A = \begin{pmatrix} 4 & 0 \\ 1 & 1 \\ 4 & -2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix}$$

i

$$\begin{aligned} 3A + B^t &= 3\begin{pmatrix} 4 & 0 \\ 1 & 1 \\ 4 & -2 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix}^t \\ &= \begin{pmatrix} 12 & 0 \\ 3 & 3 \\ 12 & -6 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 1 & 2 \\ 1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 13 & 2 \\ 4 & 5 \\ 13 & -4 \end{pmatrix} \end{aligned}$$

ii

$$\begin{aligned} AB &= \begin{pmatrix} 4 & 0 \\ 1 & 1 \\ 4 & -2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 4 & 4 & 4 \\ 3 & 3 & 3 \\ 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

2. We form the following matrix

$$\left(\begin{array}{ccc|ccc} -5 & 1 & 4 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ -4 & 1 & 3 & 0 & 0 & 1 \end{array}\right)$$

We now apply row operations to this 3×6 matrix $[A|I_n]$ to obtain the row equivalent *row-reduced echelon matrix* $[I_n|A^{-1}]$

$$\underline{\begin{array}{c} R_1 \leftrightarrow R_2 \\ (-1)R_1 \end{array}} \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & -1 & 0 \\ -5 & 1 & 4 & 1 & 0 & 0 \\ -4 & 1 & 3 & 0 & 0 & 1 \end{array}\right)$$

$$\underline{\begin{array}{c} R_2 + 5R_1 \\ R_3 + 4R_1 \end{array}} \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & -1 & 0 \\ 0 & -4 & -1 & 1 & -5 & 0 \\ 0 & -3 & -1 & 0 & -4 & 1 \end{array}\right)$$

$$\underline{R_2 - R_3} \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 & -1 & -1 \\ 0 & -3 & -1 & 0 & -4 & 1 \end{array}\right)$$

$$\underline{R_3 - 3R_2} \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 & -1 & -1 \\ 0 & 0 & -1 & -3 & -1 & 4 \end{array}\right)$$

$$\underline{\begin{array}{c} (-1)R_2 \\ (-1)R_3 \end{array}} \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 3 & 1 & -4 \end{array}\right)$$

$$\underline{R_1 + R_3} \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 3 & 0 & -4 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 3 & 1 & -4 \end{array}\right)$$

$$\underline{R_1 + R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & -3 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 3 & 1 & -4 \end{array}\right)$$

Therefore, we have

$$A^{-1} = \begin{pmatrix} 2 & 1 & -3 \\ -1 & 1 & 1 \\ 3 & 1 & -4 \end{pmatrix}$$

To confirm our solution

$$A.A^{-1} = \begin{pmatrix} -5 & 1 & 4 \\ -1 & 1 & 1 \\ -4 & 1 & 3 \end{pmatrix} . \begin{pmatrix} 2 & 1 & -3 \\ -1 & 1 & 1 \\ 3 & 1 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

3. Rotate the triangle ABC whose vertices are the points $A(0,0)$, $B(2,3)$, $C(4,1)$

i 90^o *anti-clockwise* about the origin,

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$T[A] = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = A'$$

$$T[B] = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} = B'$$

$$T[C] = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} = C'$$

ii 180^o *anti-clockwise* about the origin.

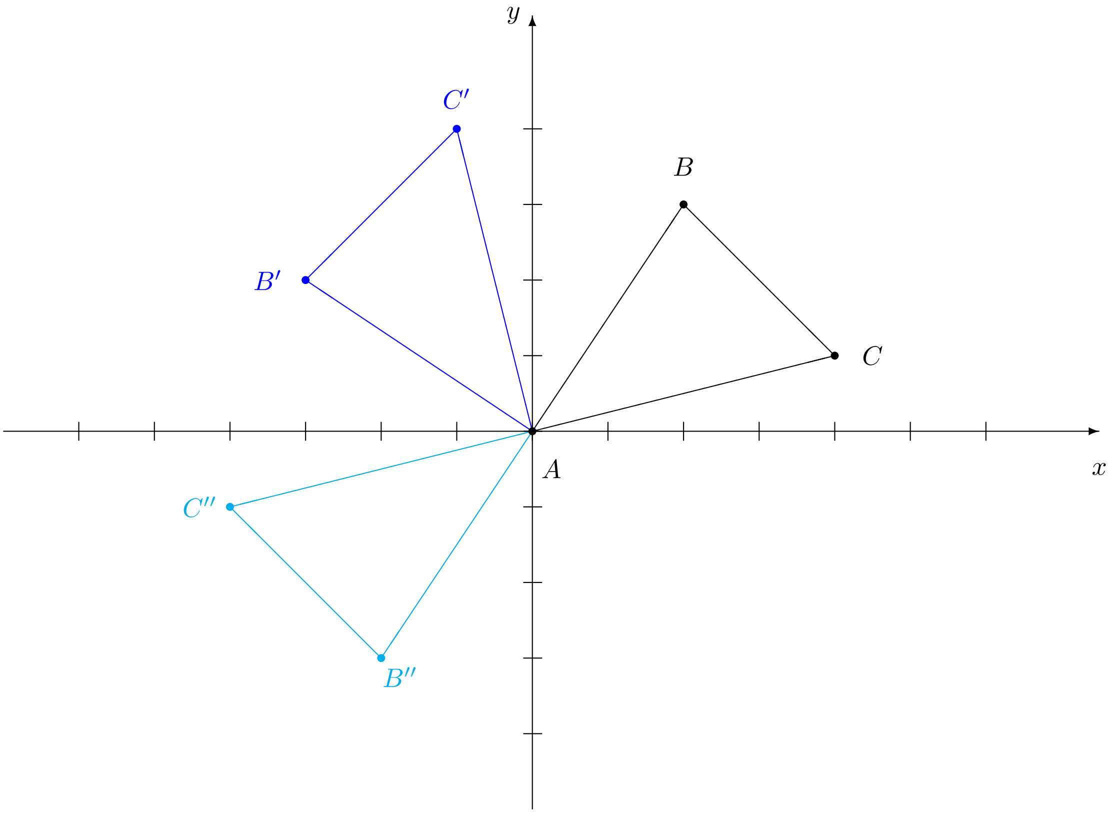
$$A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$T[A] = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = A''$$

$$T[B] = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix} = B''$$

$$T[C] = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} = C''$$

To illustrate each rotation on a simple graph.



An *orthogonal* matrix is one where the *standard matrix* A satisfies the condition $A^{-1} = A^t$. The *standard matrix* required to rotate a point *anti-clockwise* about the origin in $\mathbb{R}^2$ will be *orthogonal*. This will ensure that magnitudes and angles will be preserved as a result of the transformation. Choose just **ONE** of the matrices from part i or ii above and show that it is *orthogonal*.

For a transformation of 90^o *anti-clockwise* about the origin, we have

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Now

$$A^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Also

$$A^t = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Hence $A^{-1} = A^t$, i.e., A is *orthogonal*.