Institiúid Teicneolaíochta Cheatharlach
INSTITUTE of TECHNOLOGY CARLOW
At the Heart of South Leinster

## INSTRUCTIONS

Full marks will be awarded for the correct solutions to **ALL QUESTIONS**. This paper will be marked out of a TOTAL MAXIMUM MARK OF 100. Credit will be given for clearly presented solutions.

**BSc (Hons) in Computer Games Development**

**YEAR 1**

**CLASS TEST 3**

**1.** (a) Given the *quaternions*

$$
\begin{aligned}
q_1 &= 1 + 4i + 2j - k \\
q_2 &= 2 - 4i + 5j + 2k
\end{aligned}
$$

Evaluate each of the following

i $3q_1 + q_2$,

ii $q_1 \times q_2$,

**20 marks**

(b) Consider the point $r(2, 2, 2)$ in $\mathbb{R}^3$.

Rotate this point $180^o$ *anti-clockwise* about the y-axis, using a quaternion. (i.e., about the vector $\vec{v} = (0, 1, 0)$)

**Note**: Full workings must be shown for this question.

**20 marks**

**2.** Consider a line $\Omega$ through the point $(-2, 1, 5)$ that is *perpendicular* to the plane $\Pi$ with equation $4x - 2y + 2z + 1 = 0$.

(a) Find the parametric equations of the line $\Omega$.

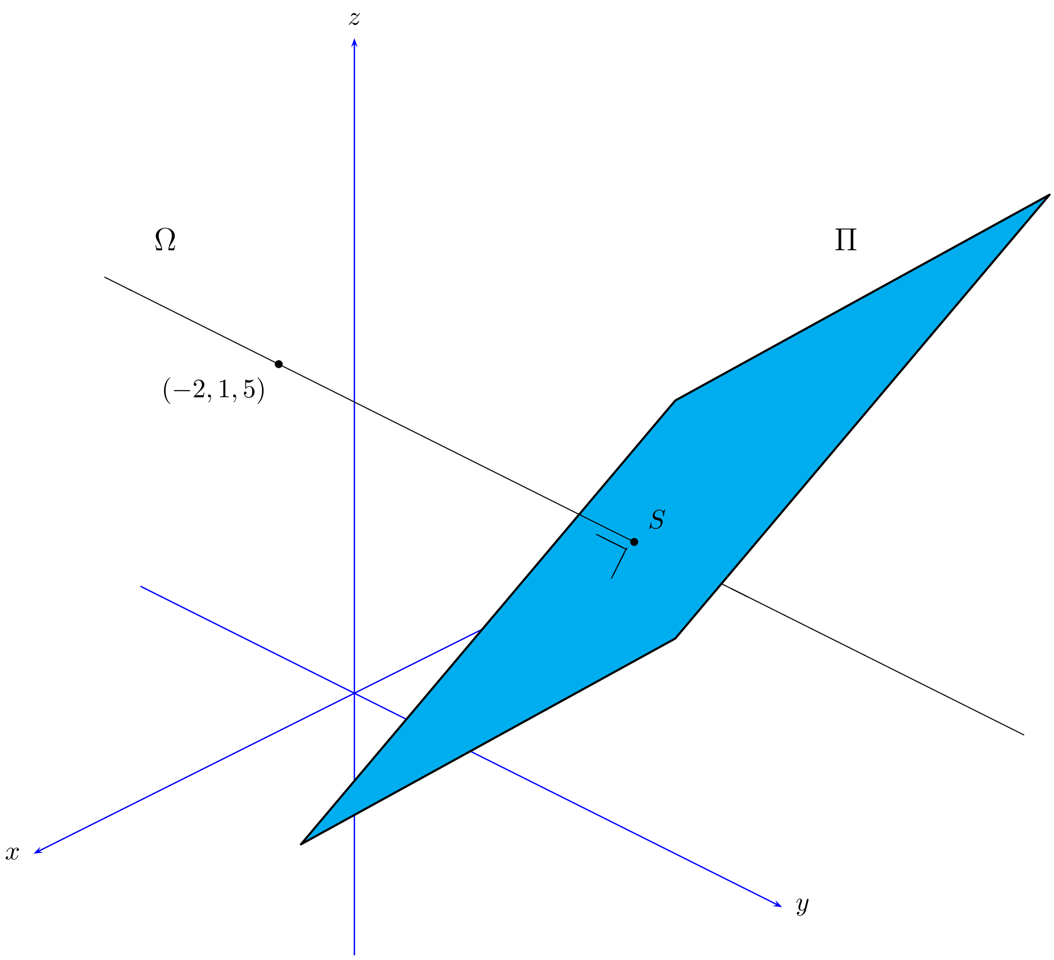
**25 marks**

(b) Find the point $S$ at which $\Omega$ meets $\Pi$.

**20 marks**

(c) Determine the perpendicular distance from the point $T(1, 2, 3)$ to the plane $\Pi$.

**15 marks**



**LINES AND PLANES IN $\mathbb{R}^3$:**

Let $\Omega$ be a **line** in $\mathbb{R}^3$.

i Find a point $P_0 = (x_0, y_0, z_0)$ which is on $\Omega$.

ii Find a vector $\vec{v} = (v_1, v_2, v_3)$ which is parallel to $\Omega$.

Then $t\vec{v}$ is also parallel to $\Omega$ and the line is the set of all points $P = (x, y, z)$ for which the vector

$$\overrightarrow{P_0P} = (x - x_0, y - y_0, z - z_0)$$

is equal to $t\vec{v}$, or

$$\overrightarrow{P_0P} = t\vec{v}$$

for some $t \in \mathbb{R}$.

———o———

Let $\Pi$ be a **plane** in $\mathbb{R}^3$.

i Find the co-ordinates of a point $P_0 = (x_0, y_0, z_0)$ which is in the plane.

ii Find a vector $\vec{n} = (a, b, c)$ perpendicular to the plane.

Then the plane $\Pi$ consists of those points $P = (x, y, z)$ for which the vector

$$\overrightarrow{P_0P} = (x - x_0, y - y_0, z - z_0)$$

is orthogonal to $\vec{n}$, or

$$\overrightarrow{P_0P}.\vec{n} = 0$$

———o———

**Theorem 1** *The perpendicular distance $D$ of a point $P = (x_0, y_0, z_0)$ from the plane $ax + by + cz + d = 0$ is given by*

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

———o———

## QUATERNIONS:

From the formula defining multiplication of quaternions we have that

$$i^2 = j^2 = k^2 = -1$$

$$ij = -ji = k \quad , \quad jk = -kj = i \quad , \quad ki = -ik = j$$

If follows directly from these identities that $ijk = -1$. The operation of multiplication on the set $\mathbb{H}$ of quaternions is **not** commutative.

---

## QUATERNIONS AND ROTATIONS:

The following quaternion

$$q = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}(li + mj + nk)$$

will rotate a point $r(x, y, z)$ through $\theta$ *anti-clockwise* about the unit vector $\vec{n} = (l, m, n)$. The rotation will be achieved by **Rotation** = $\mathbf{q}.\mathbf{r}.\mathbf{q^{-1}}$ where $r(x, y, z) = xi + yj + zk$.

Note also that for a quaternion $q = w + xi + yj + zk$

$$q^{-1} = \frac{1}{|q|^2}\bar{q}$$

the modulus is given as $|q|^2 = w^2 + x^2 + y^2 + z^2$ and the conjugate is given as $\bar{q} = w - xi - yj - zk$

---

**NAME:** ______________________________