

INSTRUCTIONS

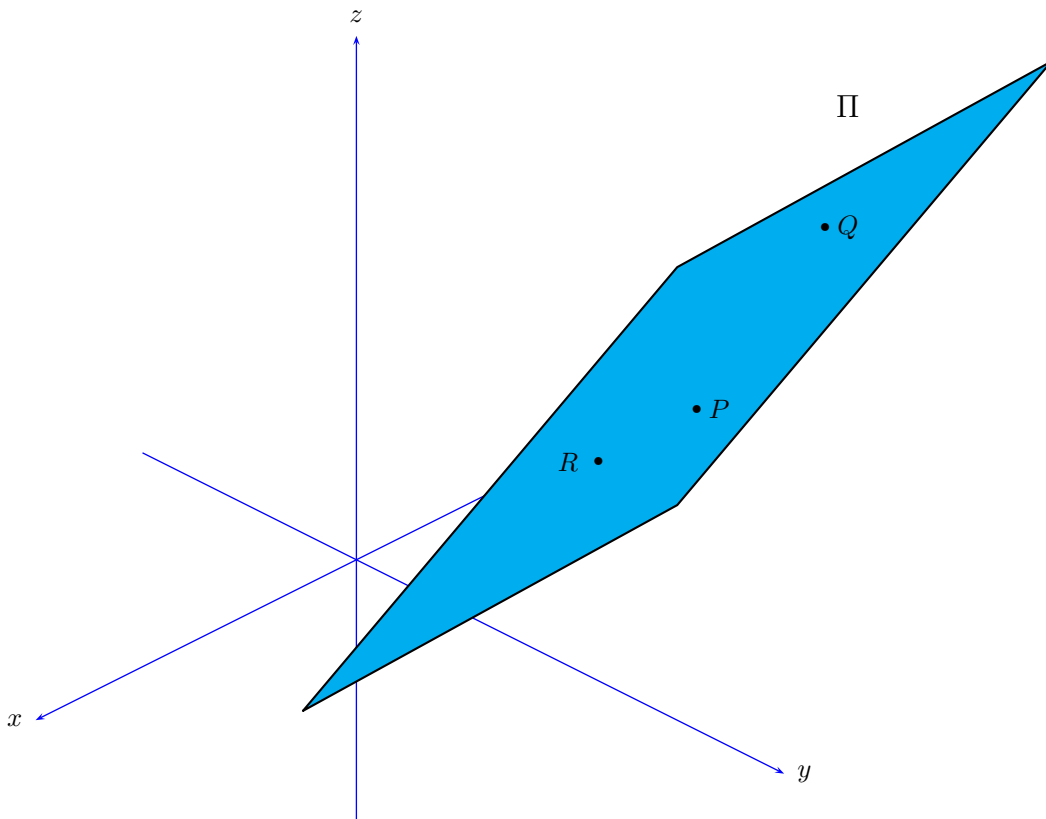
Full marks will be awarded for the correct solutions to **ALL QUESTIONS**. This paper will be marked out of a **TOTAL MAXIMUM MARK OF 100**. Credit will be given for clearly presented solutions.

BSc (Hons) in Computer Games Development
YEAR 1
CLASS TEST 3 (SOLUTIONS)

1. Consider the following points in \mathbb{R}^3

$$P(1, 1, -1) \quad , \quad Q(2, 2, 2) \quad , \quad R(3, 2, 0)$$

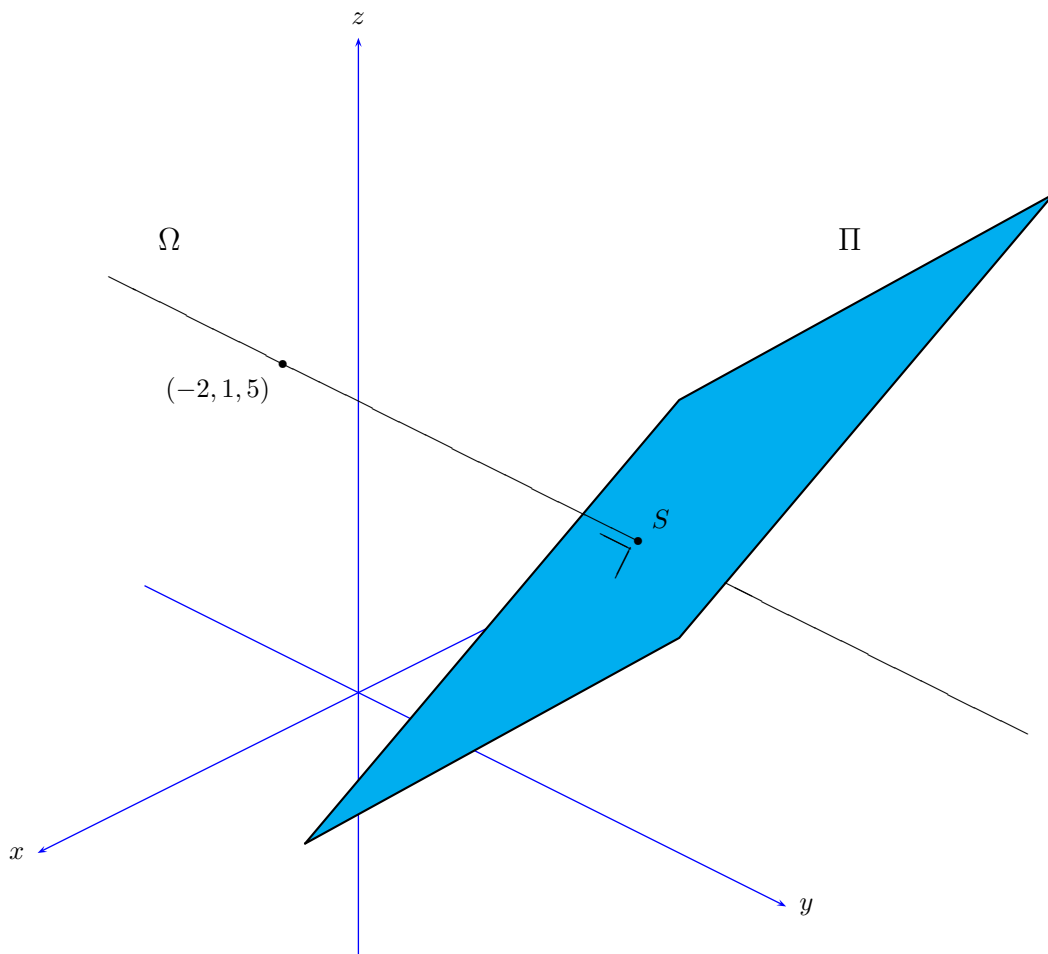
Find the equation of the plane Π that contain these points.



40 marks

2. Consider a line Ω through the point $(-2, 1, 5)$ that is *perpendicular* to the plane Π with equation $4x - 2y + 2z + 1 = 0$.

- i Find the parametric equations of the line Ω .
- ii Find the point S at which Ω meets Π .
- iii Determine the perpendicular distance from the point $T(1, 2, 3)$ to the plane Π .



60 marks

SOLUTIONS

1. $P(1, 1, -1)$, $Q(2, 2, 2)$ and $R(3, 2, 0)$ are points on the plane Π .

Firstly, we require a normal vector to Π .

$$\begin{aligned}\overrightarrow{PQ} &= \vec{Q} - \vec{P} \\ &= (2, 2, 2) - (1, 1, -1) \\ &= (1, 1, 3)\end{aligned}$$

$$\begin{aligned}\overrightarrow{PR} &= \vec{R} - \vec{P} \\ &= (3, 2, 0) - (1, 1, -1) \\ &= (2, 1, 1)\end{aligned}$$

$$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \det \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix} = (-2, 5, -1)$$

Shorthand Method The equation of the plane Π is $-2x + 5y - z + d = 0$. But $P(1, 1, -1) \in \Pi$, hence $-2(1) + 5(1) - (-1) + d = 0$. Therefore $d = -4$. Finally the equation of the plane Π is

$$-2x + 5y - z - 4 = 0$$

Alternative Solution

Let $P_0 = (1, 1, -1)$ which is on the plane.

Let $\vec{n} = (-2, 5, -1)$ perpendicular to the plane.

Then the plane consists of those points $P = (x, y, z)$ for which the vector

$$\overrightarrow{P_0P} = P - P_0 = (x - 1, y - 1, z + 1)$$

is orthogonal to \vec{n} i.e.,

$$\overrightarrow{P_0P} \cdot \vec{n} = 0$$

So that the plane has implicit equation

$$-2(x - 1) + 5(y - 1) - 1(z + 1) = 0$$

$$\therefore -2x + 5y - z - 4 = 0$$

2. *i* The line Ω goes through the points $P_0(-2, 1, 5)$ and is perpendicular to the plane Π with equation $4x - 2y + 2z + 1 = 0$, hence $\vec{v} = (4, -2, 2)$. Now

$$\begin{aligned}\overrightarrow{P_0P} &= \vec{P} - \vec{P}_0 \\ &= (x, y, z) - (-2, 1, 5) \\ &= (x + 2, y - 1, z - 5)\end{aligned}$$

Now $\overrightarrow{P_0P} = t\vec{v}$, hence

$$\begin{aligned}(x + 2, y - 1, z - 5) &= t(4, -2, 2) \\ &= (4t, -2t, 2t)\end{aligned}$$

for some $t \in \mathbb{R}$. Expanding gives the parametric equations of the line Ω .

$$\begin{aligned}x &= -2 + 4t \\ y &= 1 - 2t \\ z &= 5 + 2t\end{aligned}$$

- ii* Now to calculate the point of intersection of the line Ω and the plane

$$\Pi: 4x - 2y + 2z + 1 = 0.$$

$$\begin{aligned}4(-2 + 4t) - 2(1 - 2t) + 2(5 + 2t) + 1 &= 0 \\ -8 + 16t - 2 + 4t + 10 + 4t + 1 &= 0 \\ 24t &= -1 \\ t &= -\frac{1}{24}\end{aligned}$$

Hence, the point of intersection is

$$\begin{aligned}x &= -2 + 4\left(-\frac{1}{24}\right) = -\frac{13}{6} \\ y &= 1 - 2\left(-\frac{1}{24}\right) = \frac{13}{12} \\ z &= 5 + 2\left(-\frac{1}{24}\right) = \frac{59}{12}\end{aligned}$$

Therefore, $P = \left(-\frac{13}{6}, \frac{13}{12}, \frac{59}{12}\right)$.

- iii* Finally, the perpendicular distance D of the point $T(1, 2, 3)$ from the plane Π

$4x - 2y + 2z + 1 = 0$ is given as

$$D = \frac{|4(1) - 2(2) + 2(3) + 1|}{\sqrt{4^2 + (-2)^2 + 2^2}} = \frac{7}{\sqrt{24}}$$