INSTRUCTIONS

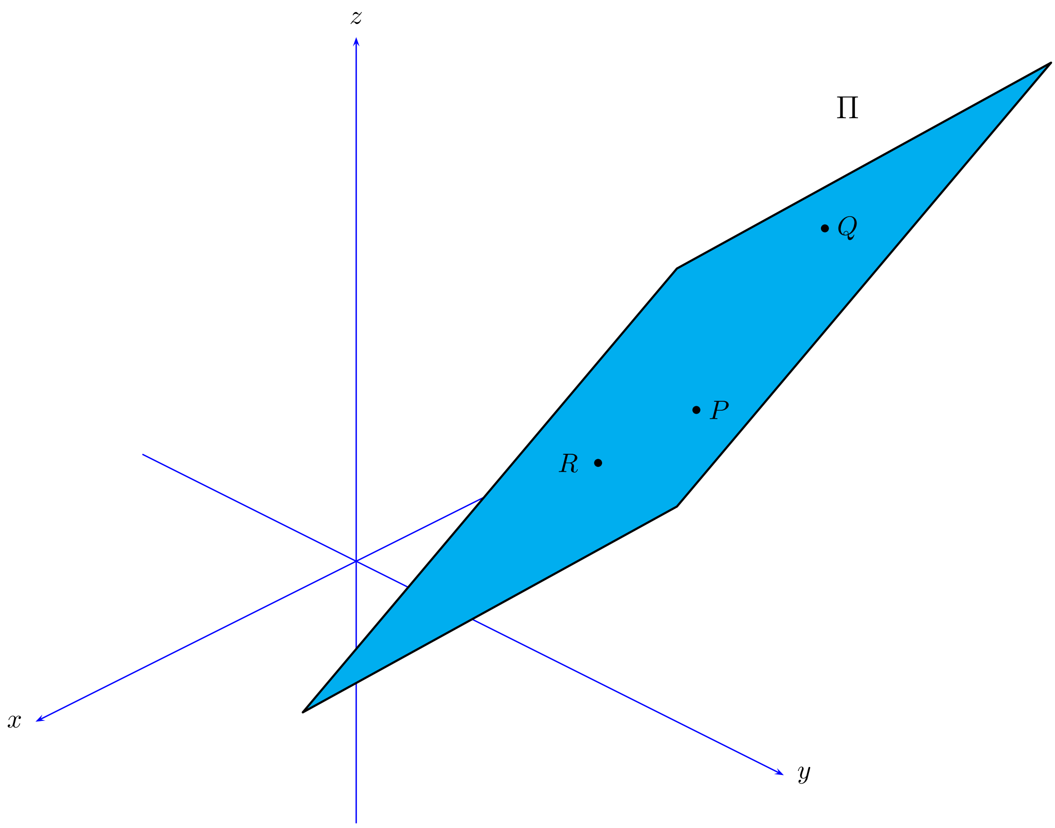
Full marks will be awarded for the correct solutions to **ALL QUESTIONS**. This paper will be marked out of a TOTAL MAXIMUM MARK OF 100. Credit will be given for clearly presented solutions.

BSc (Hons) in Computer Games Development
YEAR 1
CLASS TEST 3 (SOLUTIONS)

1. Consider the following points in $\mathbb{R}^3$

$$P(1,1,-1) \quad , \quad Q(2,2,2) \quad , \quad R(3,2,0)$$

Find the equation of the plane Π that contain these points.



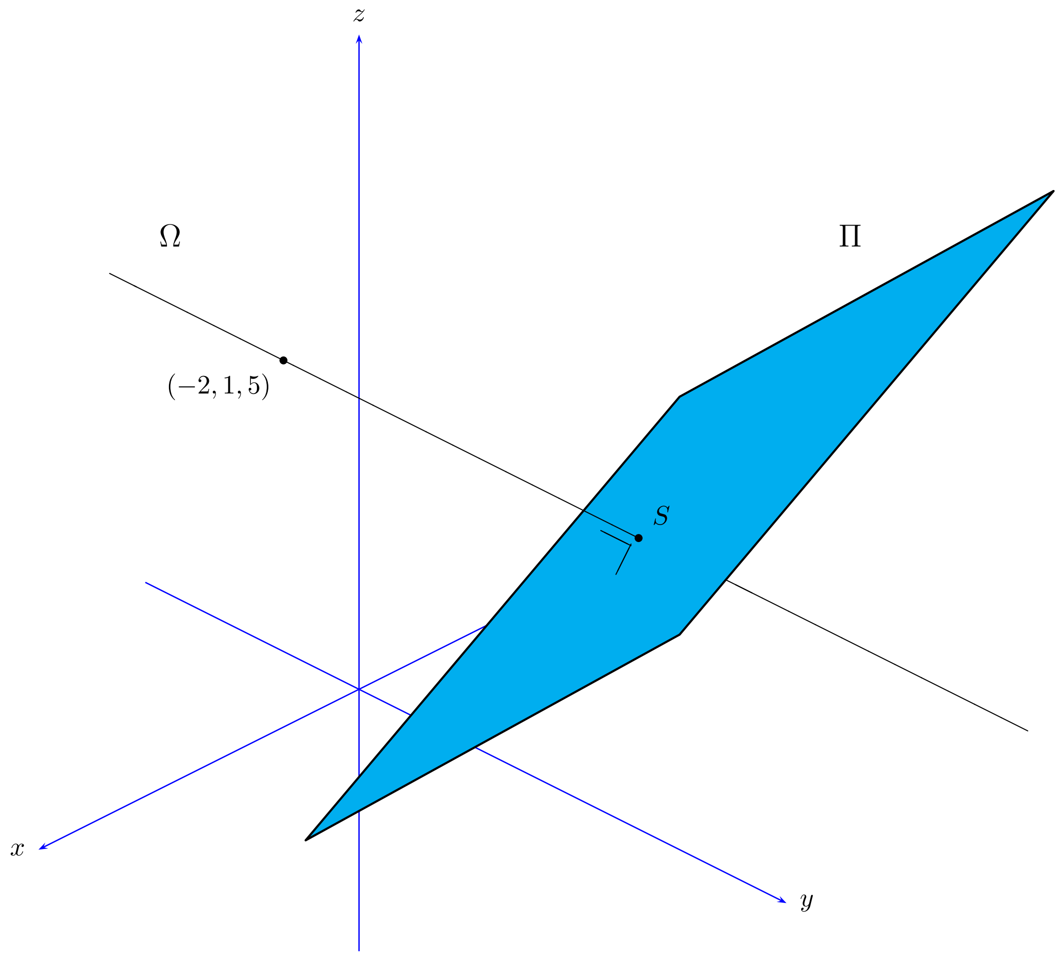
40 marks

2. Consider a line Ω through the point $(-2, 1, 5)$ that is *perpendicular* to the plane Π with equation $4x - 2y + 2z + 1 = 0$.

i Find the parametric equations of the line Ω.

ii Find the point S at which Ω meets Π.

iii Determine the perpendicular distance from the point $T(1, 2, 3)$ to the plane Π.



60 marks

SOLUTIONS

1. *$P(1,1,-1)$, $Q(2,2,2)$ and $R(3,2,0)$ are points on the* **plane** Π.

Firstly, we require a normal vector to Π.

$$\begin{aligned}\overrightarrow{PQ} &= \vec{Q} - \vec{P}\\ &= (2,2,2)-(1,1,-1)\\ &= (1,1,3)\end{aligned}$$

$$\begin{aligned}\overrightarrow{PR} &= \vec{R} - \vec{P}\\ &= (3,2,0)-(1,1,-1)\\ &= (2,1,1)\end{aligned}$$

$$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = det\begin{vmatrix} \vec{i} & \vec{j} & \vec{k}\\ 1 & 1 & 3\\ 2 & 1 & 1\end{vmatrix} = (-2,5,-1)$$

Shorthand Method *The equation of the plane Π is $-2x+5y-z+d=0$. But $P(1,1,-1) \in \Pi$, hence $-2(1)+5(1)-(-1)+d=0$. Therefore $d=-4$. Finally the equation of the plane Π is*

$$-2x+5y-z-4=0$$

Alternative Solution

Let $P_0 = (1,1,-1)$ which is on the plane.

Let $\vec{n} = (-2,5,-1)$ perpendicular to the plane.

Then the plane consists of those points $P=(x,y,z)$ for which the vector

$$\overrightarrow{P_0P} = P - P_0 = (x-1, y-1, z+1)$$

is orthogonal to $\vec{n}$ i.e.,

$$\overrightarrow{P_0P}.\vec{n} = 0$$

So that the plane has implicit equation

$$-2(x-1)+5(y-1)-1(z+1)=0$$

$$\therefore \quad -2x+5y-z-4=0$$

2. *i The line Ω goes through the points $P_0(-2,1,5)$ and is perpendicular to the plane Π with equation $4x-2y+2z+1=0$, hence $\vec{v}=(4,-2,2)$. Now*

$$\begin{aligned}\overrightarrow{P_0P} &= \vec{P}-\vec{P_0}\\ &= (x,y,z)-(-2,1,5)\\ &= (x+2,y-1,z-5)\end{aligned}$$

Now $\overrightarrow{P_0P}=t\vec{v}$, hence

$$\begin{aligned}(x+2,y-1,z-5) &= t(4,-2,2)\\ &= (4t,-2t,2t)\end{aligned}$$

for some $t\in\mathbb{R}$. Expanding gives the parametric equations of the line Ω.

$$\begin{aligned}x &= -2+4t\\ y &= 1-2t\\ z &= 5+2t\end{aligned}$$

ii Now to calculate the point of intersection of the line Ω and the plane $\Pi:\ 4x-2y+2z+1=0$.

$$\begin{aligned}4(-2+4t)-2(1-2t)+2(5+2t)+1 &= 0\\ -8+16t-2+4t+10+4t+1 &= 0\\ 24t &= -1\\ t &= -\frac{1}{24}\end{aligned}$$

Hence, the point of intersection is

$$\begin{aligned}x &= -2+4(-\frac{1}{24}) = -\frac{13}{6}\\ y &= 1-2(-\frac{1}{24}) = \frac{13}{12}\\ z &= 5+2(-\frac{1}{24}) = \frac{59}{12}\end{aligned}$$

Therefore, $P=(-\frac{13}{6},\frac{13}{12},\frac{59}{12})$.

iii Finally, the perpendicular distance D of the point $T(1,2,3)$ from the plane Π $4x-2y+2z+1=0$ is given as

$$D=\frac{|4(1)-2(2)+2(3)+1|}{\sqrt{4^2+(-2)^2+2^2}}=\frac{7}{\sqrt{24}}$$