

# Functional Blocks

COE 202

Digital Logic Design

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# Presentation Outline

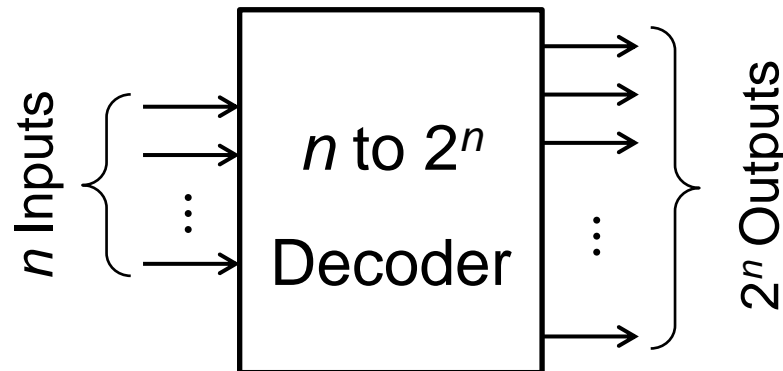
- ❖ Decoders
- ❖ Encoders
- ❖ Multiplexers
- ❖ Demultiplexers
- ❖ Design Examples

# Functional Blocks

- ❖ A functional block is a combinational circuit
- ❖ We will study blocks, such as decoders and multiplexers
- ❖ Functional blocks are very common and useful in design
- ❖ In the past, functional blocks were integrated circuits
  - SSI:** Small Scale Integration = tens of gates
  - MSI:** Medium Scale Integration = hundreds of gates
  - LSI:** Large Scale Integration = thousands of gates
  - VLSI:** Very Large Scale Integration = millions of gates
- ❖ Today, functional blocks are part of a design library
- ❖ Tested for correctness and reused in many projects

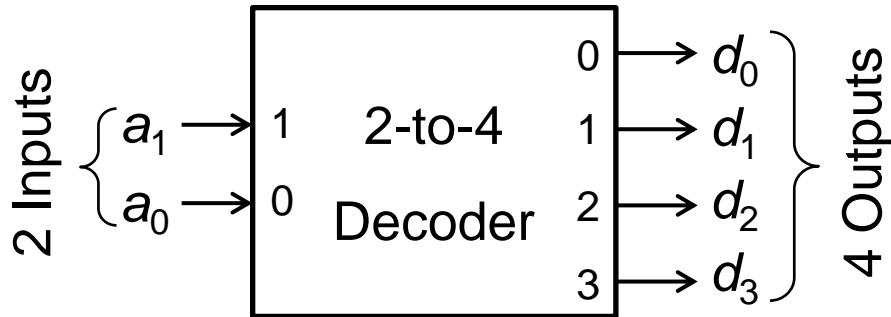
# Binary Decoders

- ❖ Given a  $n$ -bit binary code, there are  $2^n$  possible code values
- ❖ The decoder has an output for each possible code value
- ❖ The  $n$ -to- $2^n$  decoder has  $n$  inputs and  $2^n$  outputs
- ❖ Depending on the input code, **only one output** is set to **logic 1**
- ❖ The conversion of input to output is called **decoding**



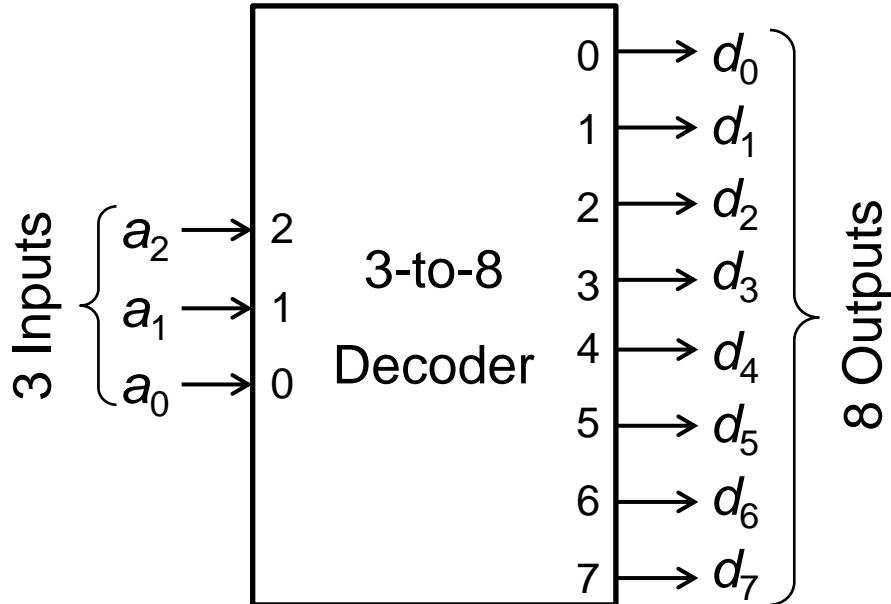
A decoder can have less than  $2^n$  outputs if some input codes are unused

# Examples of Binary Decoders



Inputs		Outputs			
$a_1$	$a_0$	$d_0$	$d_1$	$d_2$	$d_3$
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

**Truth  
Tables**

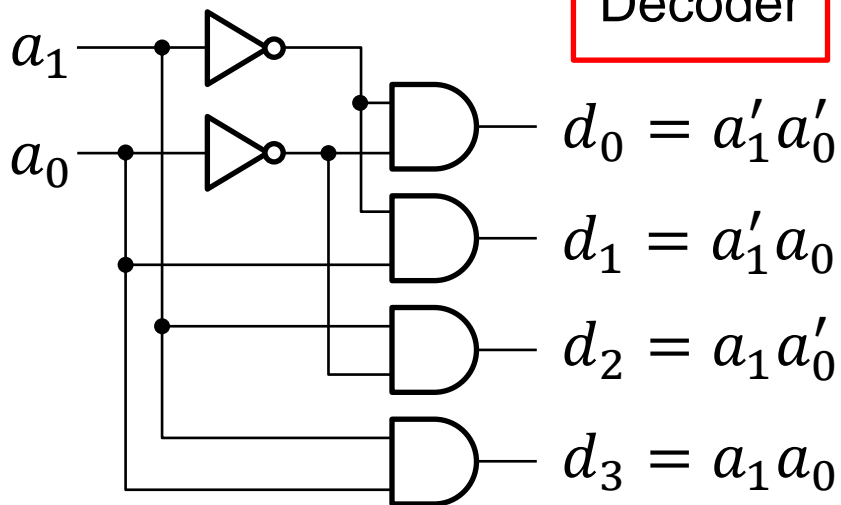


Inputs			Outputs							
$a_2$	$a_1$	$a_0$	$d_0$	$d_1$	$d_2$	$d_3$	$d_4$	$d_5$	$d_6$	$d_7$
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

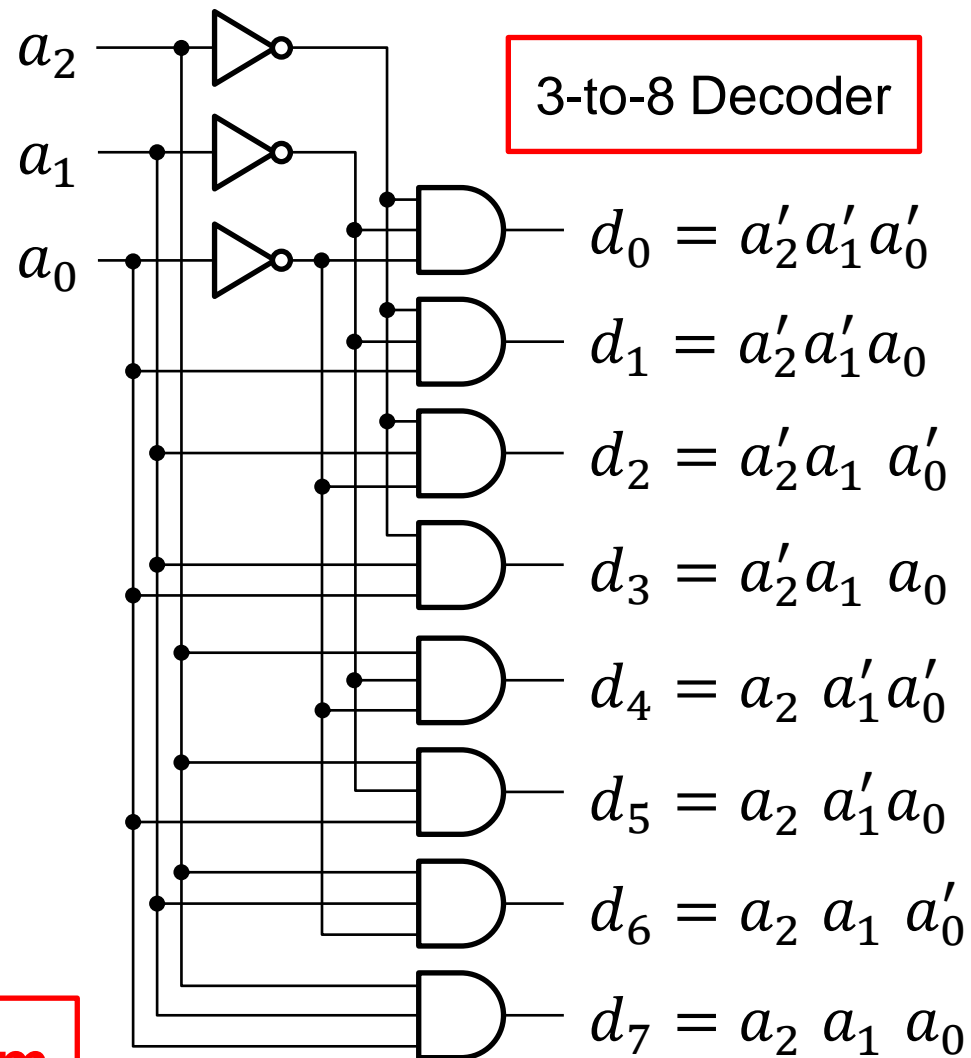
# Decoder Implementation

Inputs		Outputs			
$a_1$	$a_0$	$d_0$	$d_1$	$d_2$	$d_3$
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

2-to-4 Decoder



Each decoder output is a **minterm**



3-to-8 Decoder

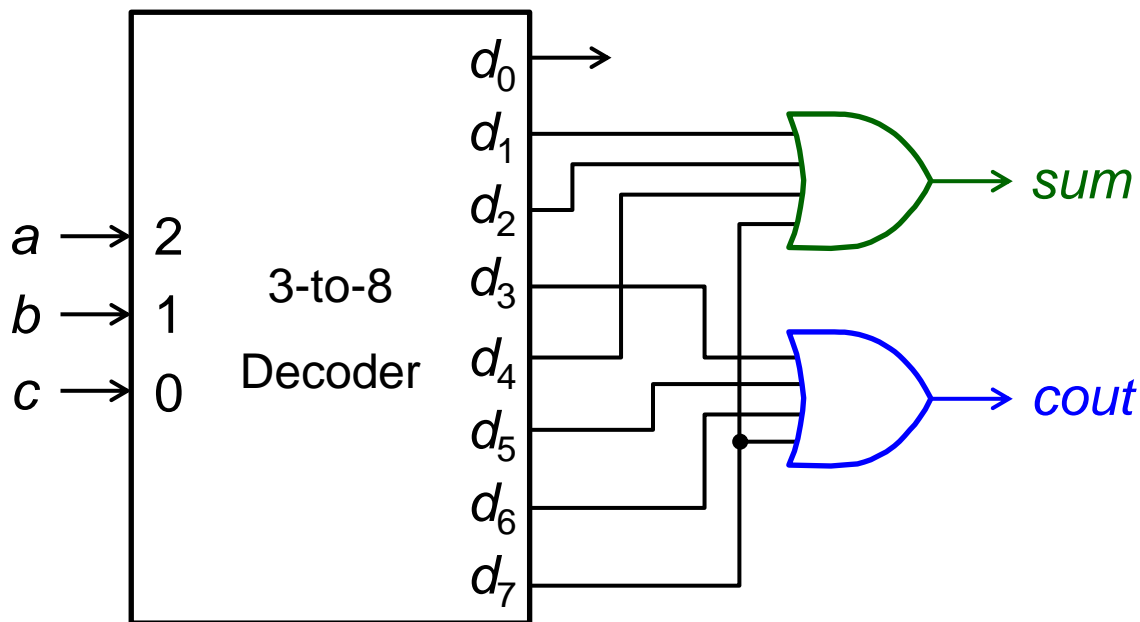
# Using Decoders to Implement Functions

- ❖ A decoder generates all the minterms
- ❖ A Boolean function can be expressed as a sum of minterms
- ❖ Any function can be implemented using a decoder + OR gate

Note: the function **must not be minimized**

- ❖ **Example:** Full Adder  $sum = \Sigma(1, 2, 4, 7)$ ,  $cout = \Sigma(3, 5, 6, 7)$

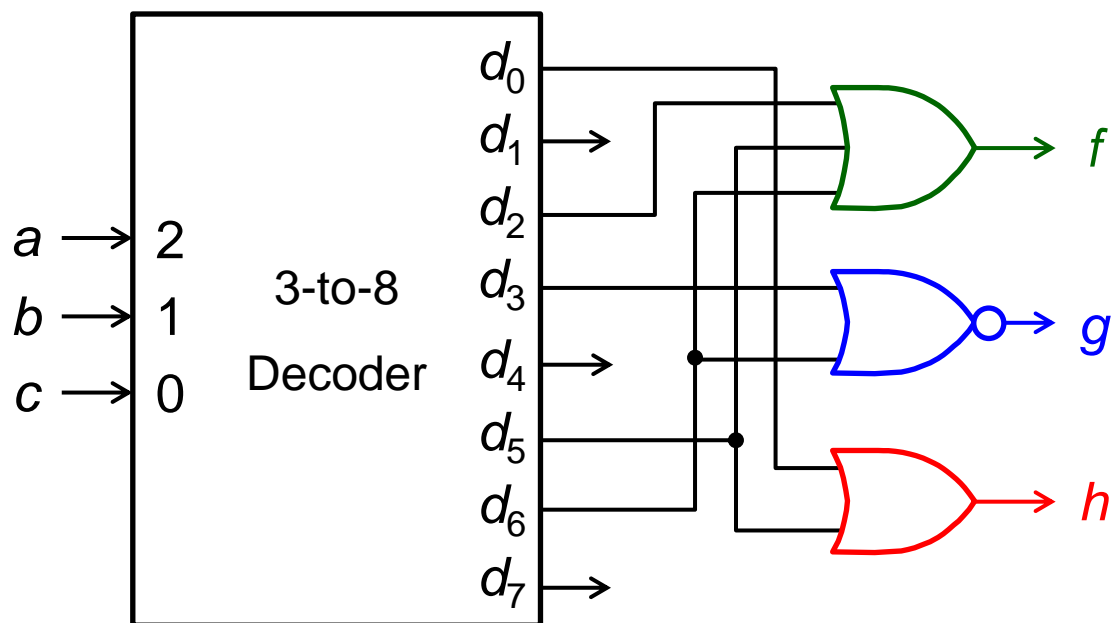
Inputs			Outputs	
a	b	c	cout	sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



# Using Decoders to Implement Functions

- ❖ Good if many output functions of the same input variables
- ❖ If number of minterms is large → Wider OR gate is needed
- ❖ Use NOR gate if number of maxterms is less than minterms
- ❖ **Example:**  $f = \Sigma(2, 5, 6)$ ,  $g = \Pi(3, 6) \rightarrow g' = \Sigma(3, 6)$ ,  $h = \Sigma(0, 5)$

Inputs			Outputs		
a	b	c	f	g	h
0	0	0	0	1	1
0	0	1	0	1	0
0	1	0	1	1	0
0	1	1	0	0	0
1	0	0	0	1	0
1	0	1	1	1	1
1	1	0	1	0	0
1	1	1	0	1	0



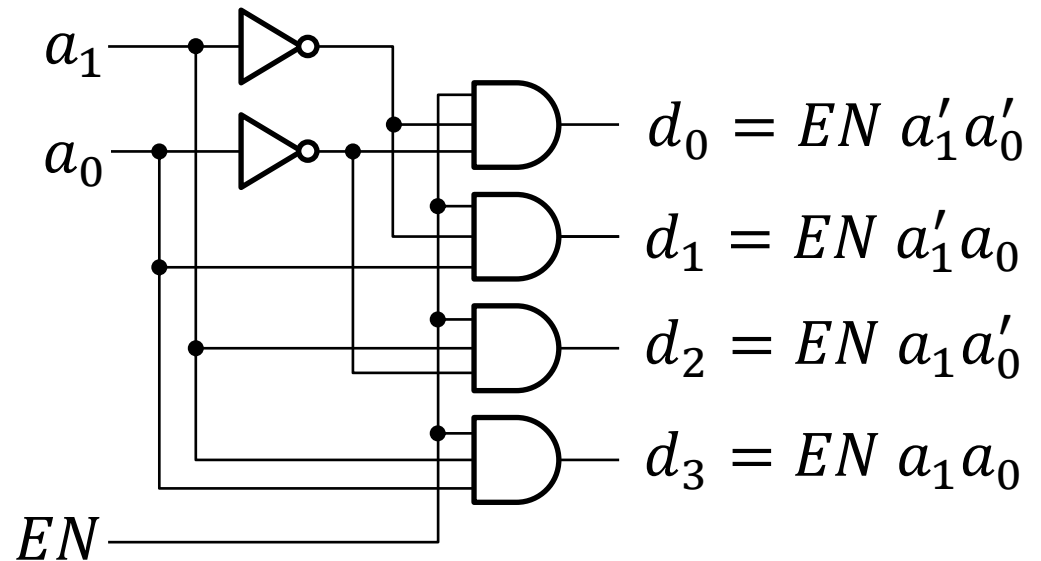
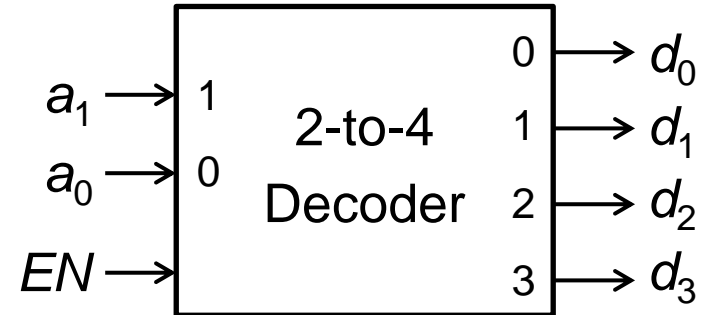


# 2-to-4 Decoder with Enable Input

## Truth Table

Inputs		Outputs			
EN	$a_1 a_0$	$d_0$	$d_1$	$d_2$	$d_3$
0	X X	0	0	0	0
1	0 0	1	0	0	0
1	0 1	0	1	0	0
1	1 0	0	0	1	0
1	1 1	0	0	0	1

If  $EN$  input is zero then all outputs are zeros, regardless of  $a_1$  and  $a_0$

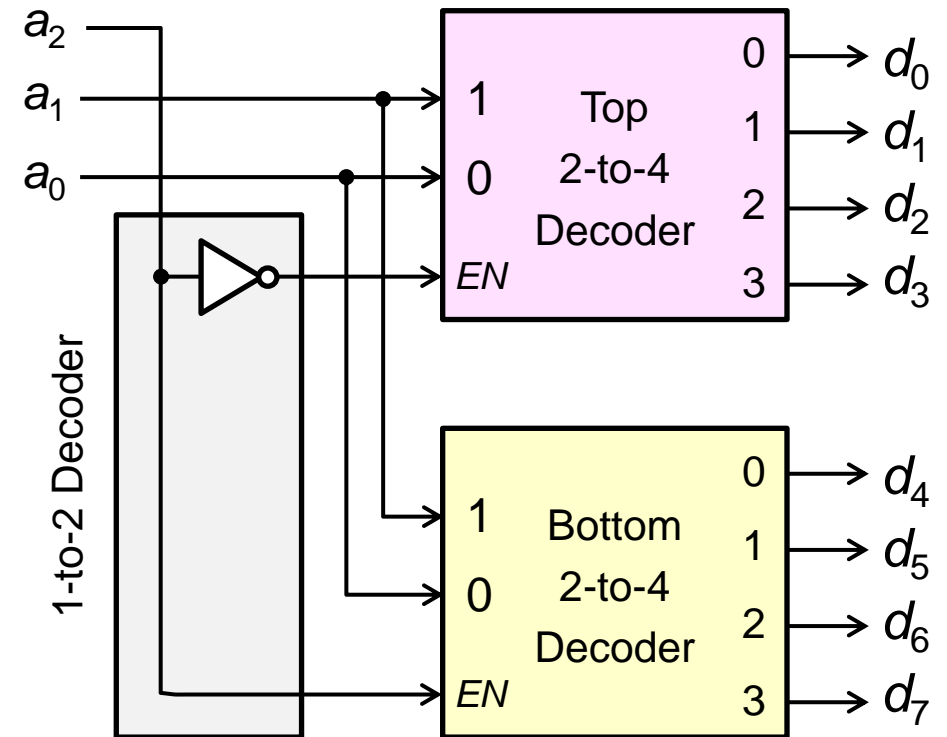


# Building Larger Decoders

- ❖ Larger decoders can be build using smaller ones
- ❖ A 3-to-8 decoder can be built using:

Two 2-to-4 decoders with Enable and an inverter (1-to-2 decoder)

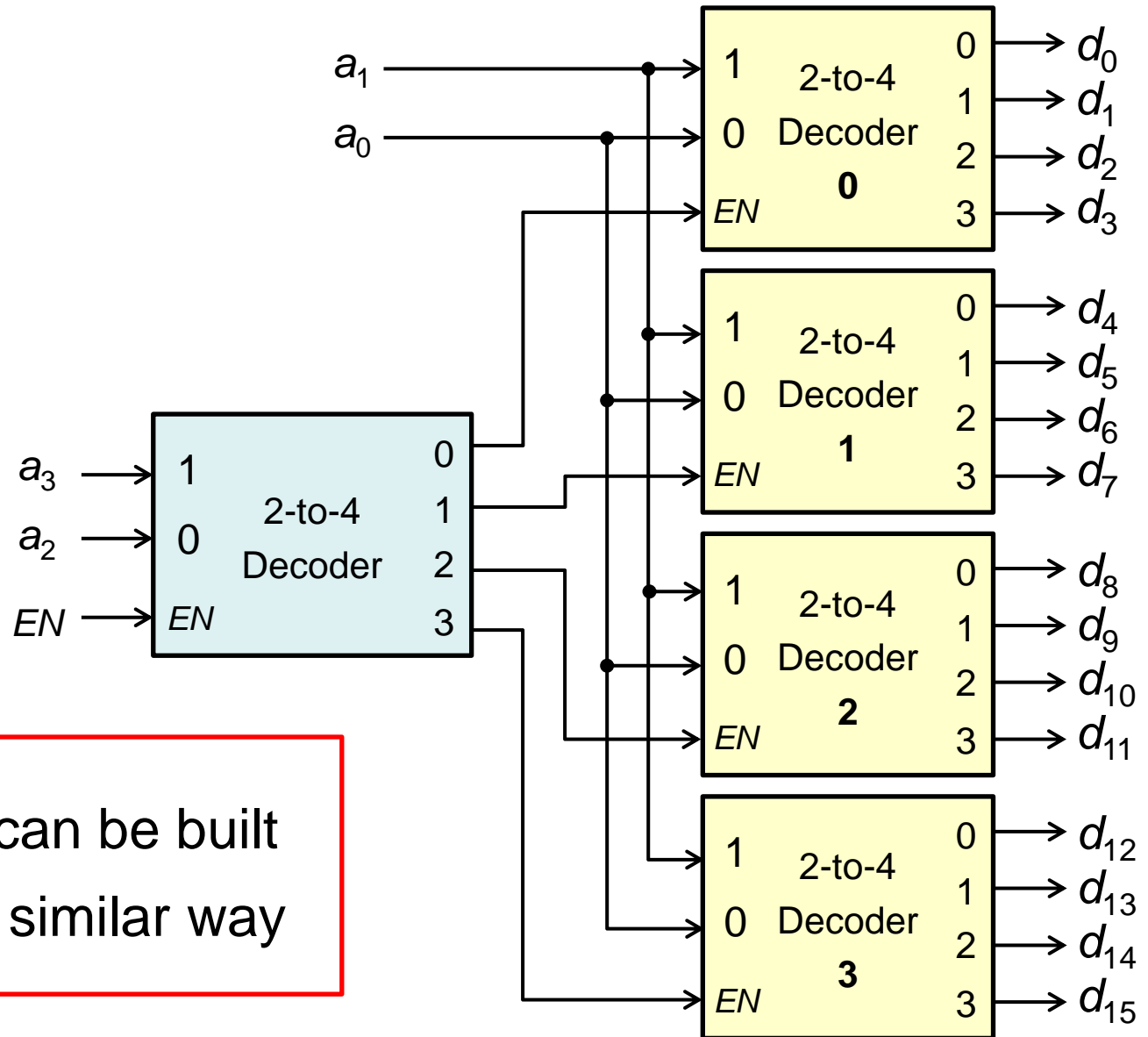
Inputs			Outputs							
$a_2$	$a_1$	$a_0$	$d_0$	$d_1$	$d_2$	$d_3$	$d_4$	$d_5$	$d_6$	$d_7$
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1



# Building Larger Decoders

A 4-to-16 decoder with enable can be built using **five** 2-to-4 decoders with enables

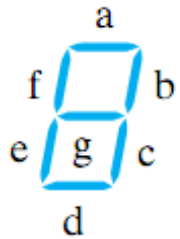
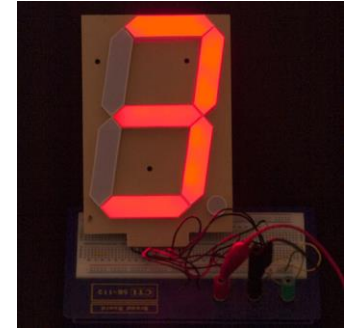
Larger decoders can be built hierarchically in a similar way



# BCD to 7-Segment Decoder

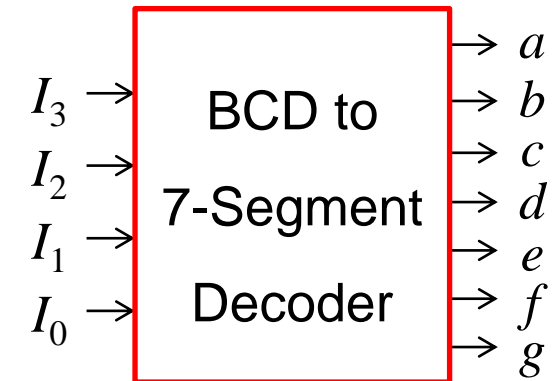
## ❖ Seven-Segment Display:

- ❖ Made of Seven segments: light-emitting diodes (LED)
- ❖ Found in electronic devices: such as clocks, calculators, etc.



## ❖ BCD to 7-Segment Decoder

- ❖ Called also a decoder, but not a binary decoder
- ❖ Accepts as input a BCD decimal digit (0 to 9)
- ❖ Generates output to the seven LED segments to display the BCD digit
- ❖ Each segment can be turned on or off separately



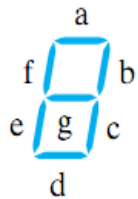
# BCD to 7-Segment Decoder

## Specification:

- ✧ Input: 4-bit BCD ( $I_3, I_2, I_1, I_0$ )
- ✧ Output: 7-bit ( $a, b, c, d, e, f, g$ )
- ✧ Display should be OFF for Non-BCD input codes.

## Implementation can use:

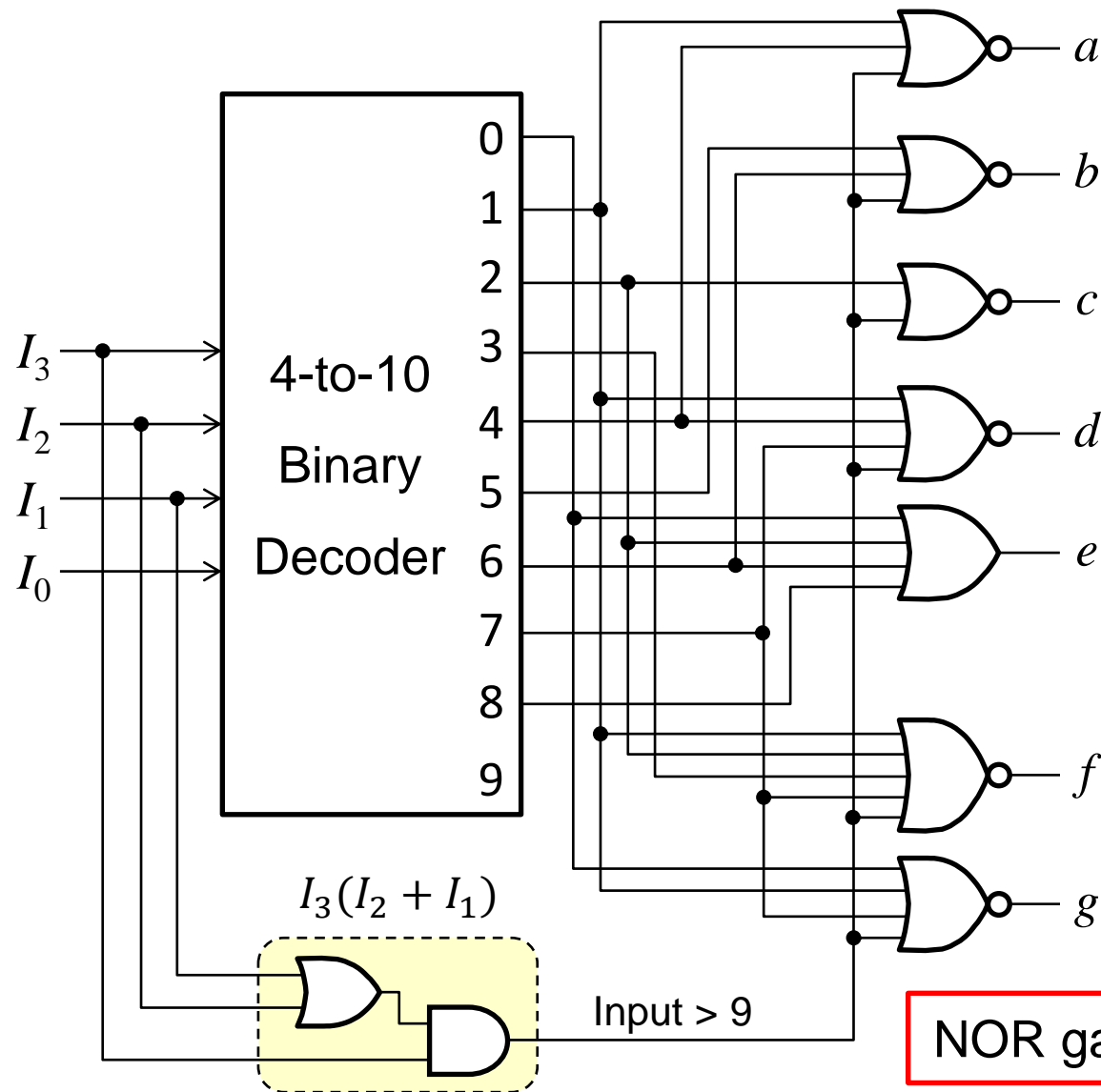
- ✧ A binary decoder
- ✧ Additional gates



## Truth Table

BCD input				7-Segment Output						
$I_3$	$I_2$	$I_1$	$I_0$	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1010 to 1111				0	0	0	0	0	0	0

# Implementing a BCD to 7-Segment Decoder



## Truth Table

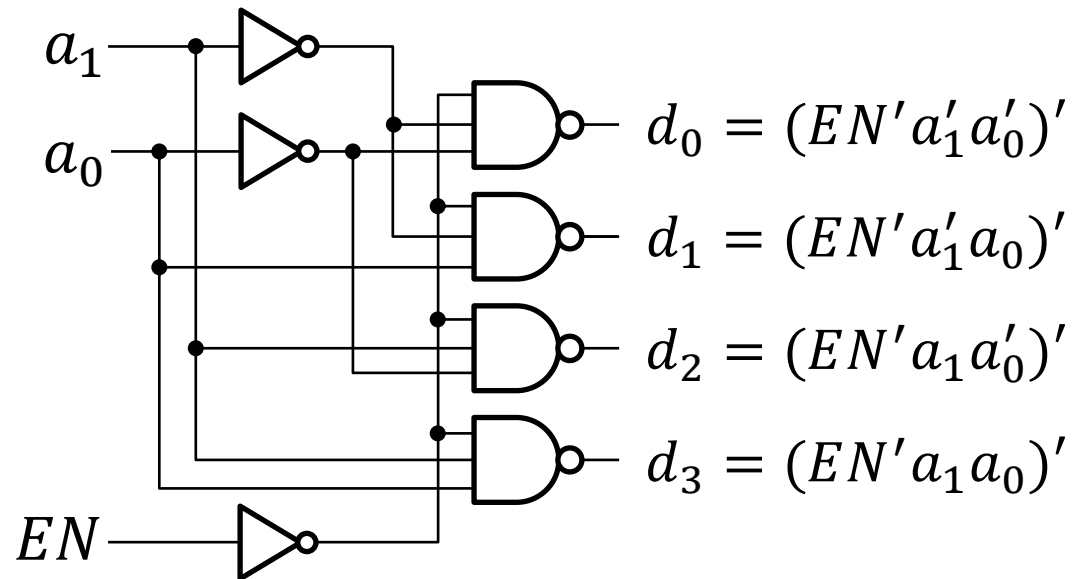
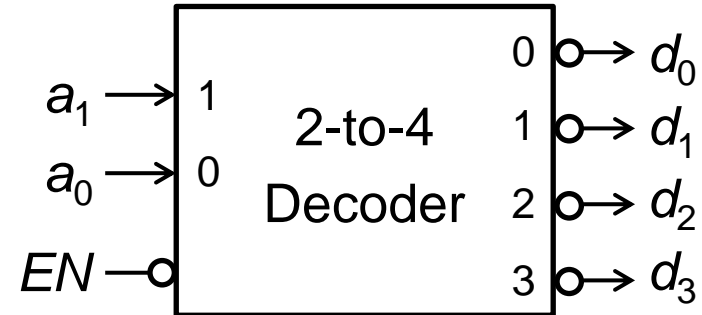
$I_3$	$I_2$	$I_1$	$I_0$	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1010	-	1111		0	0	0	0	0	0	0

NOR gate is used for 0's

# NAND Decoders with Inverted Outputs

## Truth Table

Inputs		Outputs			
EN	$a_1 a_0$	$d_0$	$d_1$	$d_2$	$d_3$
1	X X	1	1	1	1
0	0 0	0	1	1	1
0	0 1	1	0	1	1
0	1 0	1	1	0	1
0	1 1	1	1	1	0

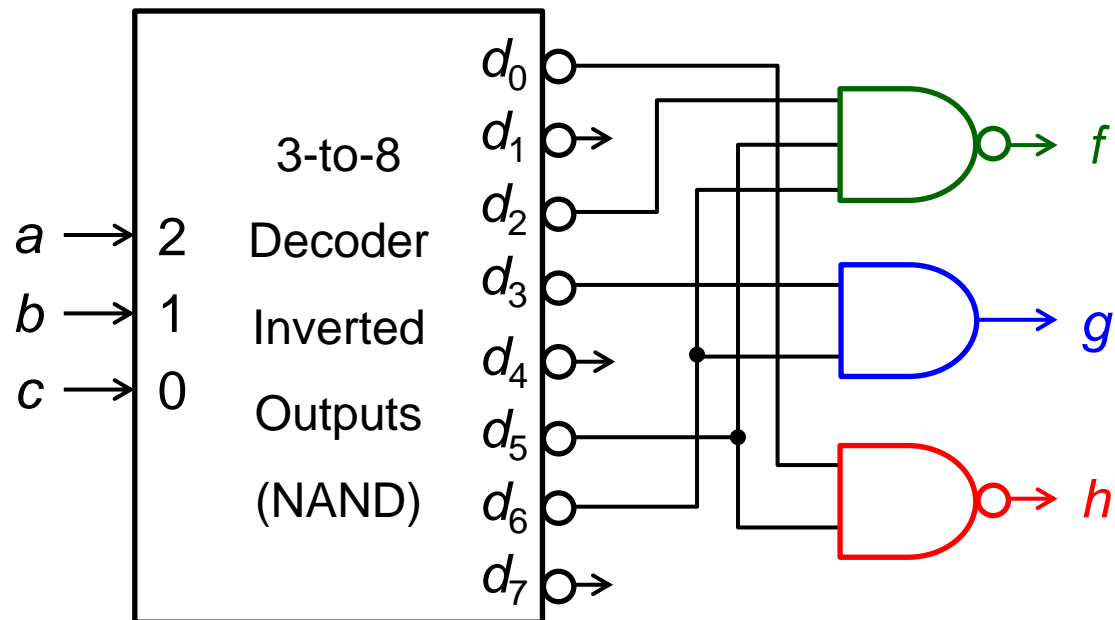


Some decoders are constructed with NAND gates. Their outputs are inverted. The Enable input is also active low (Enable if zero)

# Using NAND Decoders

- ❖ NAND decoders can be used to implement functions
- ❖ Use NAND gates to output the minterms (if fewer ones)
- ❖ Use AND gates to output the maxterms (if fewer zeros)
- ❖ **Example:**  $f = \Sigma(2, 5, 6)$ ,  $g = \Pi(3, 6)$ ,  $h = \Sigma(0, 5)$

Inputs			Outputs		
a	b	c	f	g	h
0	0	0	0	1	1
0	0	1	0	1	0
0	1	0	1	1	0
0	1	1	0	0	0
1	0	0	0	1	0
1	0	1	1	1	1
1	1	0	1	0	0
1	1	1	0	1	0





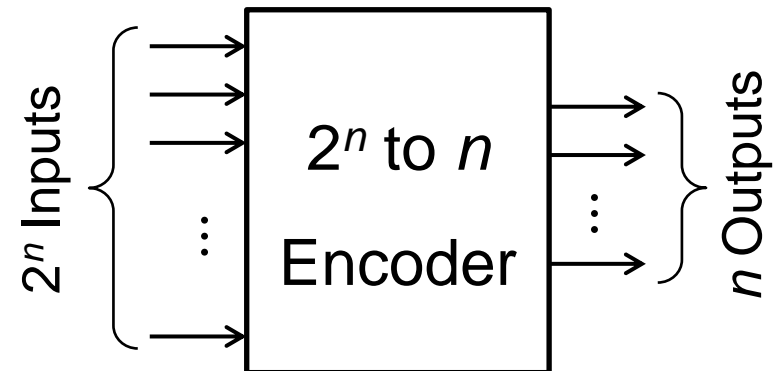
# Next . . .

- ❖ Decoders
- ❖ Encoders
- ❖ Multiplexers
- ❖ Demultiplexers
- ❖ Design Examples

# Encoders

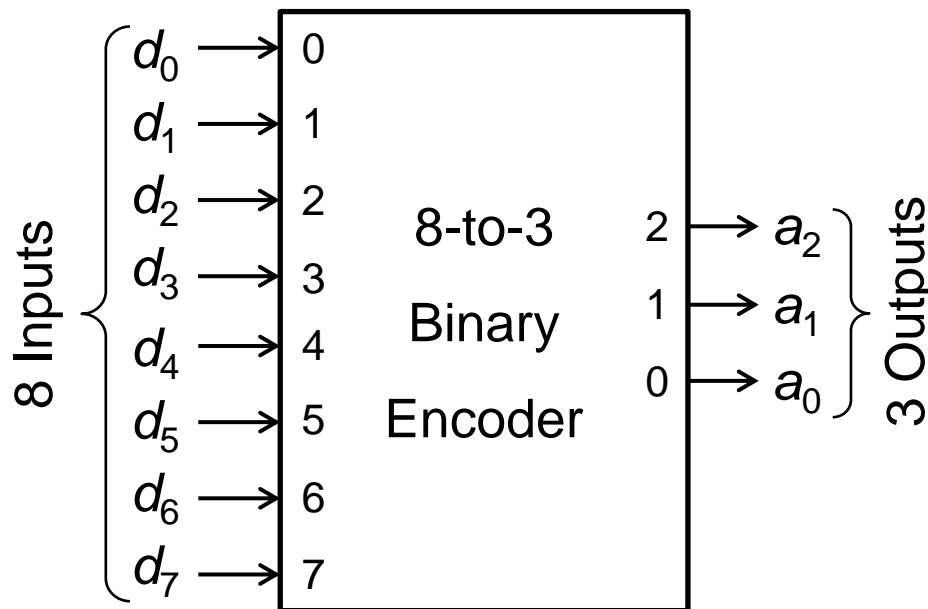
- ❖ An encoder performs the opposite operation of a decoder
- ❖ It converts a  $2^n$  input to an  $n$ -bit output code
- ❖ The output indicates which input is active (logic **1**)
- ❖ Typically, **one** input should be **1** and all others must be **0**'s
- ❖ The conversion of input to output is called **encoding**

A encoder can have less than  $2^n$  inputs if some input lines are unused



# Example of an 8-to-3 Binary Encoder

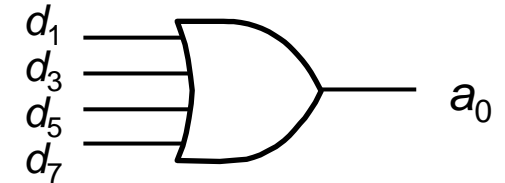
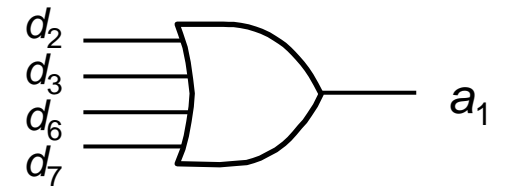
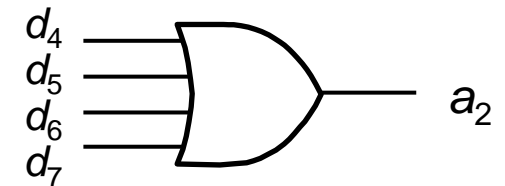
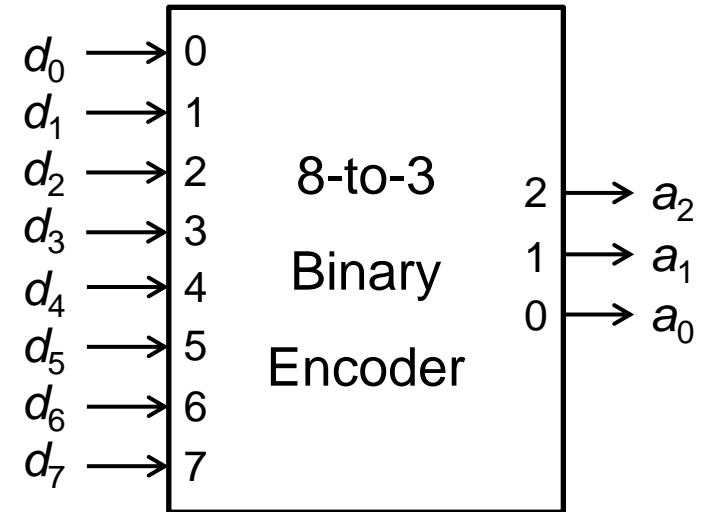
- ❖ 8 inputs, 3 outputs, only **one input** is **1**, all others are **0**'s
- ❖ Encoder generates the output binary code for the active input
- ❖ Output is **not specified** if more than one input is **1**



Inputs								Outputs		
$d_7$	$d_6$	$d_5$	$d_4$	$d_3$	$d_2$	$d_1$	$d_0$	$a_2$	$a_1$	$a_0$
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	1	1	1

# 8-to-3 Binary Encoder Implementation

Inputs								Outputs		
$d_7$	$d_6$	$d_5$	$d_4$	$d_3$	$d_2$	$d_1$	$d_0$	$a_2$	$a_1$	$a_0$
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	1	1	1



$$a_2 = d_4 + d_5 + d_6 + d_7$$

$$a_1 = d_2 + d_3 + d_6 + d_7$$

$$a_0 = d_1 + d_3 + d_5 + d_7$$

8-to-3 binary encoder implemented using three 4-input OR gates

# Binary Encoder Limitations

- ❖ Exactly **one input** must be **1** at a time (all others must be **0**'s)
- ❖ If **more than one** input is **1** then the output will be **incorrect**

- ❖ For example, if  $d_3 = d_6 = 1$

Then  $a_2 a_1 a_0 = \mathbf{111}$  (**incorrect**)

$$a_2 = d_4 + d_5 + d_6 + d_7$$

$$a_1 = d_2 + d_3 + d_6 + d_7$$

$$a_0 = d_1 + d_3 + d_5 + d_7$$

- ❖ Two problems to resolve:

1. If **two** inputs are **1** at the same time, what should be the output?
2. If **all** inputs are **0**'s, what should be the output?

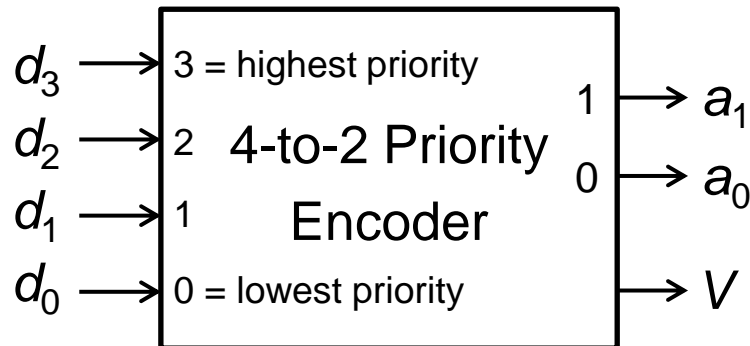
- ❖ Output  $a_2 a_1 a_0 = 000$  if  $d_0 = 1$  or all inputs are 0's

How to resolve this ambiguity?

# Priority Encoder

- ❖ Eliminates the two problems of the binary encoder
- ❖ Inputs are ranked from highest priority to lowest priority
- ❖ If **more than one** input is active (logic **1**) then priority is used  
Output encodes the active input with higher priority
- ❖ If all inputs are zeros then the **V** (Valid) output is zero

Indicates that all inputs are zeros

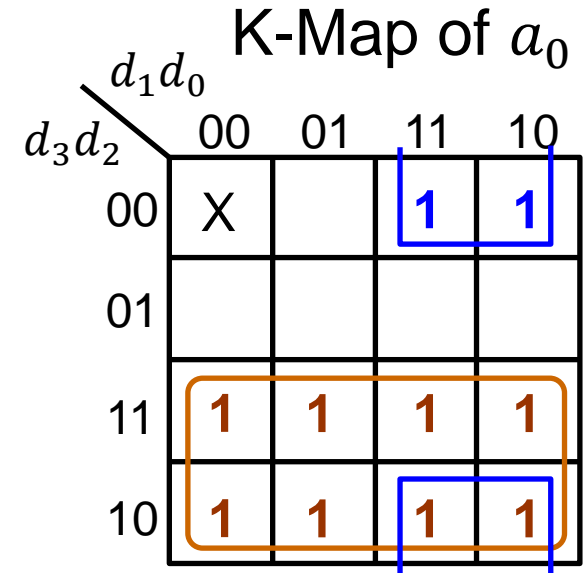
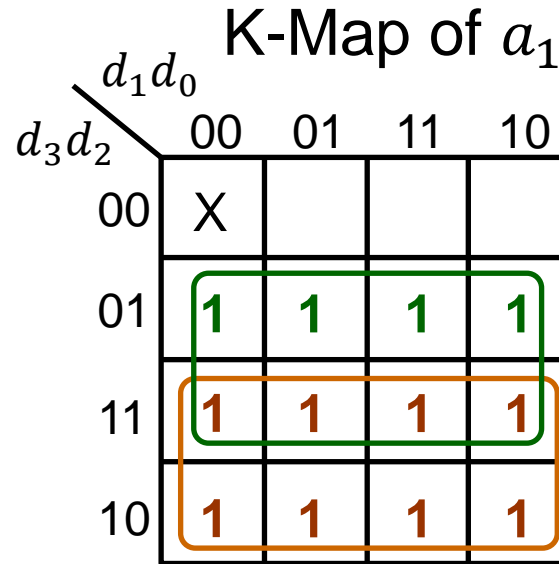


Condensed  
Truth Table  
All 16 cases  
are listed

Inputs				Outputs		
$d_3$	$d_2$	$d_1$	$d_0$	$a_1$	$a_0$	V
0	0	0	0	X	X	<b>0</b>
0	0	0	<b>1</b>	<b>0</b>	<b>0</b>	1
0	0	<b>1</b>	X	<b>0</b>	<b>1</b>	1
0	<b>1</b>	X	X	<b>1</b>	<b>0</b>	1
<b>1</b>	X	X	X	<b>1</b>	<b>1</b>	1

# Implementing a 4-to-2 Priority Encoder

Inputs				Outputs		
$d_3$	$d_2$	$d_1$	$d_0$	$a_1$	$a_0$	$V$
0	0	0	0	X	X	0
0	0	0	1	0	0	1
0	0	1	X	0	1	1
0	1	X	X	1	0	1
1	X	X	X	1	1	1

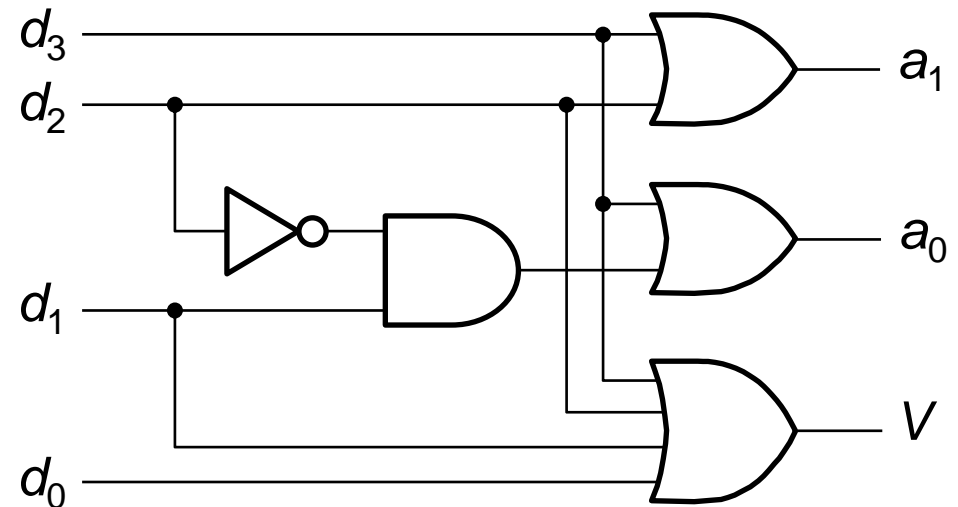


## Output Expressions:

$$a_1 = d_3 + d_2$$

$$a_0 = d_3 + d_1 d_2'$$

$$V = d_3 + d_2 + d_1 + d_0$$



# Next . . .

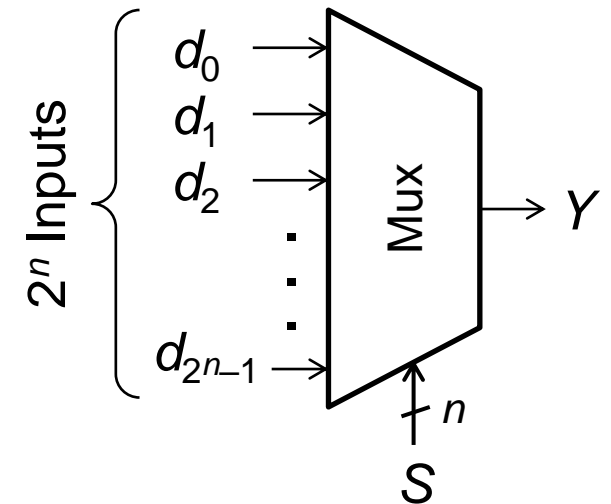
- ❖ Decoders
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# Multiplexers

- ❖ Selecting data is an essential function in digital systems
- ❖ Functional blocks that perform selecting are called **multiplexers**
- ❖ A Multiplexer (or Mux) is a combinational circuit that has:

- ✧ Multiple data inputs (typically  $2^n$ ) to select from
- ✧ An  $n$ -bit select input  $S$  used for control
- ✧ One output  $Y$



- ❖ The  $n$ -bit select input directs one of the data inputs to the output

# Examples of Multiplexers

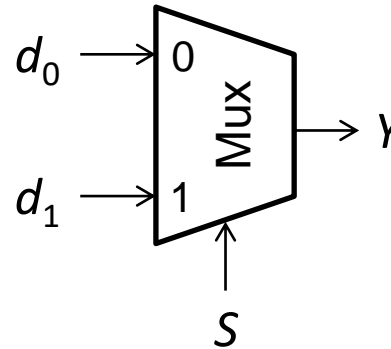
## ❖ 2-to-1 Multiplexer

if ( $S == 0$ )  $Y = d_0$ ;

else  $Y = d_1$ ;

Logic expression:

$$Y = d_0 S' + d_1 S$$



Inputs			Output
S	d <sub>0</sub>	d <sub>1</sub>	Y
0	0	X	0 = d <sub>0</sub>
0	1	X	1 = d <sub>0</sub>
1	X	0	0 = d <sub>1</sub>
1	X	1	1 = d <sub>1</sub>

## ❖ 4-to-1 Multiplexer

if ( $S_1 S_0 == 00$ )  $Y = d_0$ ;

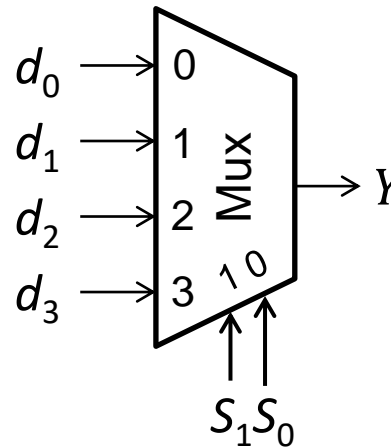
else if ( $S_1 S_0 == 01$ )  $Y = d_1$ ;

else if ( $S_1 S_0 == 10$ )  $Y = d_2$ ;

else  $Y = d_3$ ;

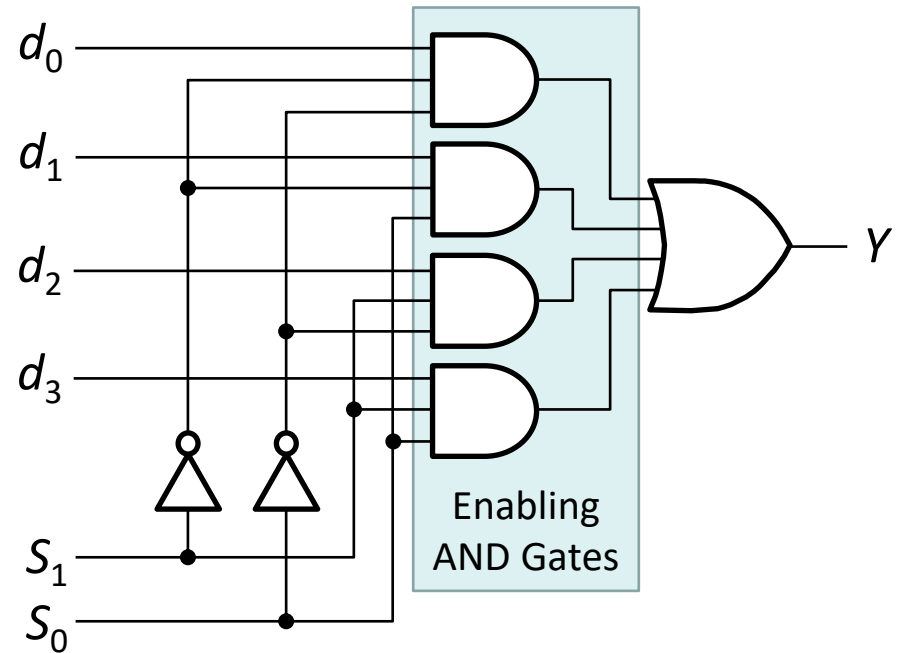
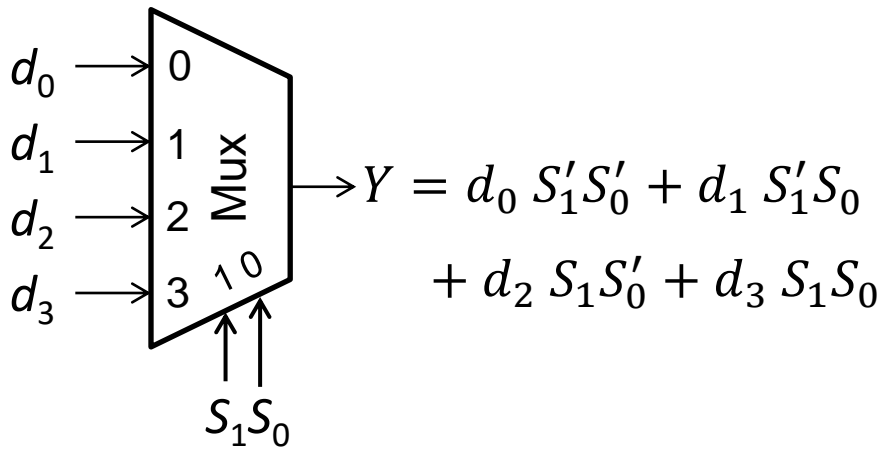
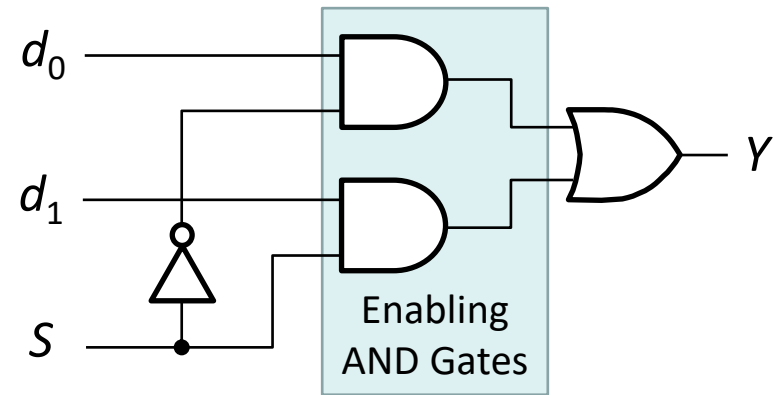
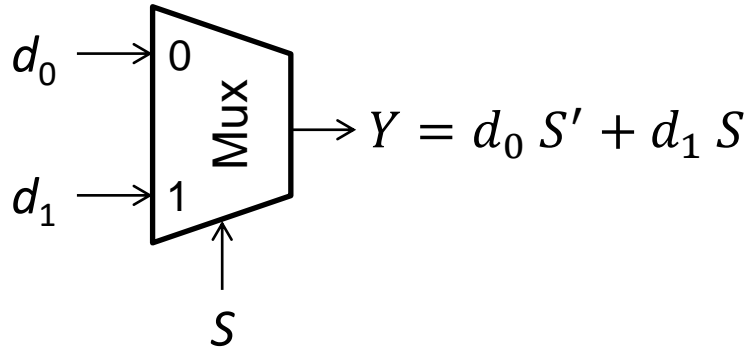
Logic expression:

$$Y = d_0 S_1' S_0' + d_1 S_1' S_0 + d_2 S_1 S_0' + d_3 S_1 S_0$$



Inputs						Output
S <sub>1</sub>	S <sub>0</sub>	d <sub>0</sub>	d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>	Y
0	0	0	X	X	X	0 = d <sub>0</sub>
0	0	1	X	X	X	1 = d <sub>0</sub>
0	1	X	0	X	X	0 = d <sub>1</sub>
0	1	X	1	X	X	1 = d <sub>1</sub>
1	0	X	X	0	X	0 = d <sub>2</sub>
1	0	X	X	1	X	1 = d <sub>2</sub>
1	1	X	X	X	0	0 = d <sub>3</sub>
1	1	X	X	X	1	1 = d <sub>3</sub>

# Implementing Multiplexers



# 3-State Gate

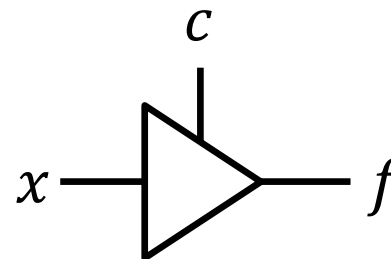
- ❖ Logic gates studied so far have two outputs: 0 and 1
- ❖ Three-State gate has three possible outputs: **0, 1, Z**
  - ✧ **Z** is the **Hi-Impedance** output
  - ✧ **Z** means that the output is **disconnected** from the input
  - ✧ Gate behaves as an **open switch** between input and output

❖ Input **c** connects input to output

✧ **c** is the control (enable) input

✧ If **c** is **0** then **f = Z**

✧ If **c** is **1** then **f = input x**

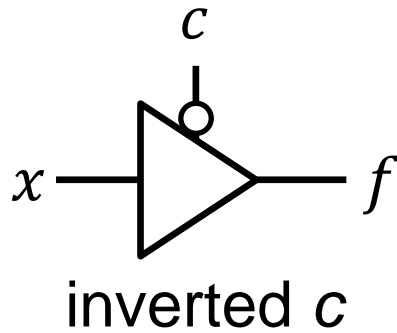


3-state gate

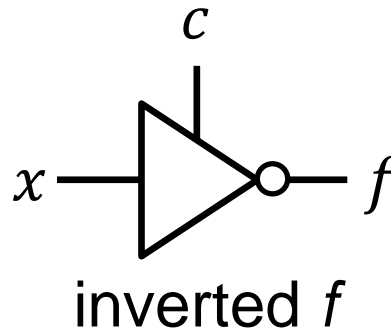
c	x	f
0	0	Z
0	1	Z
1	0	0
1	1	1

# Variations of the 3-State Gate

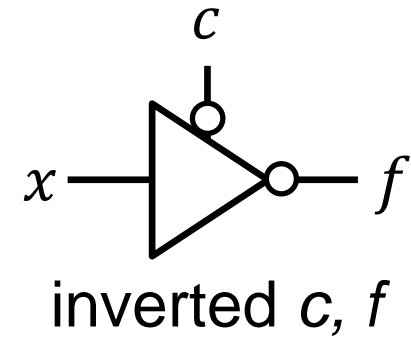
- ❖ Control input **c** and output **f** can be inverted
- ❖ A bubble is inserted at the input **c** or output **f**



<b>c</b>	<b>x</b>	<b>f</b>
0	0	0
0	1	1
1	0	Z
1	1	Z



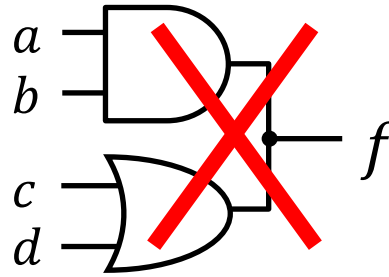
<b>c</b>	<b>x</b>	<b>f</b>
0	0	Z
0	1	Z
1	0	1
1	1	0



<b>c</b>	<b>x</b>	<b>f</b>
0	0	1
0	1	0
1	0	Z
1	1	Z

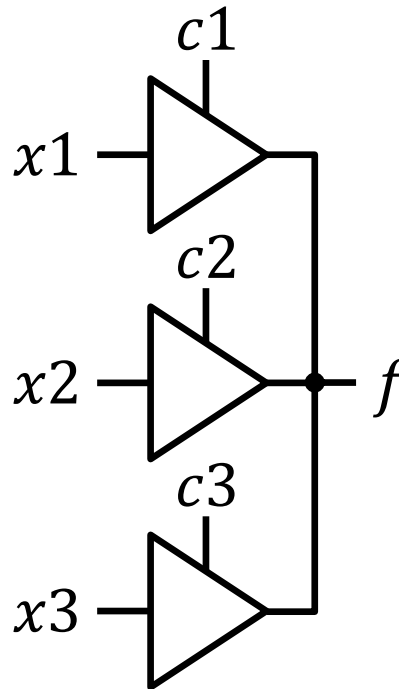
# Wired Output

Logic gates with 0 and 1 outputs **cannot** have their outputs wired together



This will result in a **short circuit** that will burn the gates

3-state gates **can wire** their outputs together



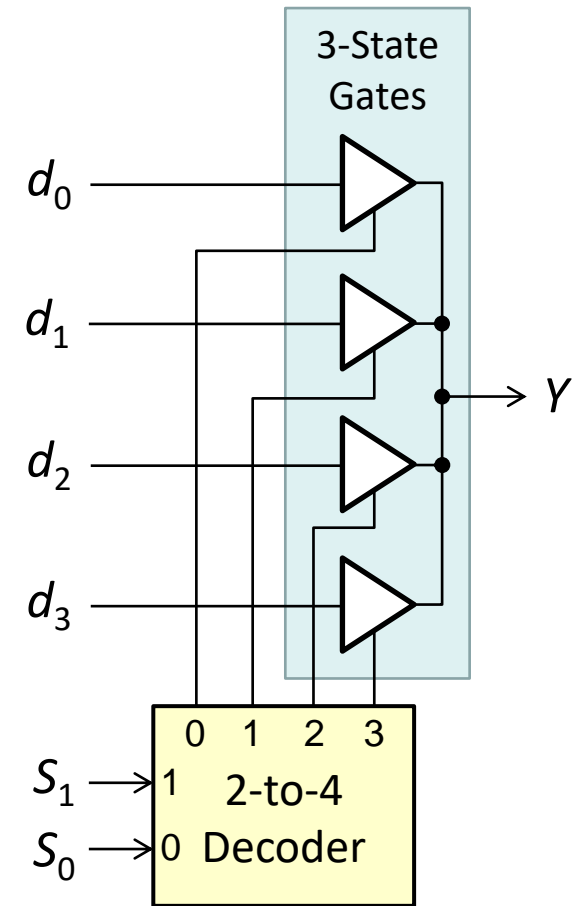
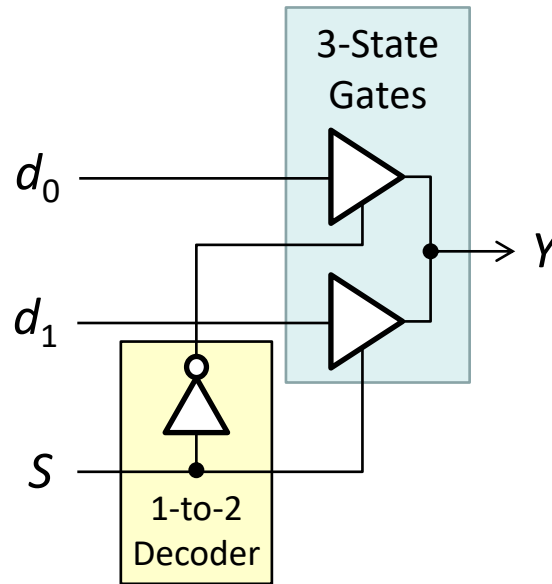
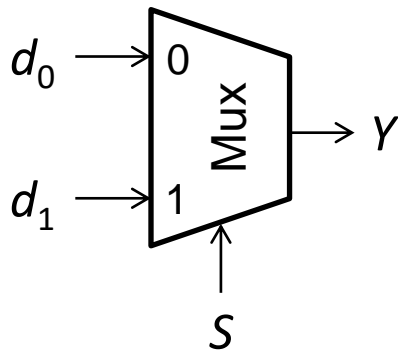
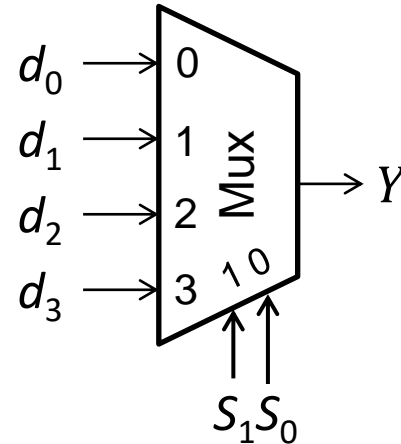
**At most one** 3-state gate can be enabled at a time  
Otherwise, conflicting outputs will burn the circuit

c1	c2	c3	f
0	0	0	z
1	0	0	x1
0	1	0	x2
0	0	1	x3
0	1	1	Burn
1	0	1	Burn
1	1	0	Burn
1	1	1	Burn

# Implementing Multiplexers with 3-State Gates

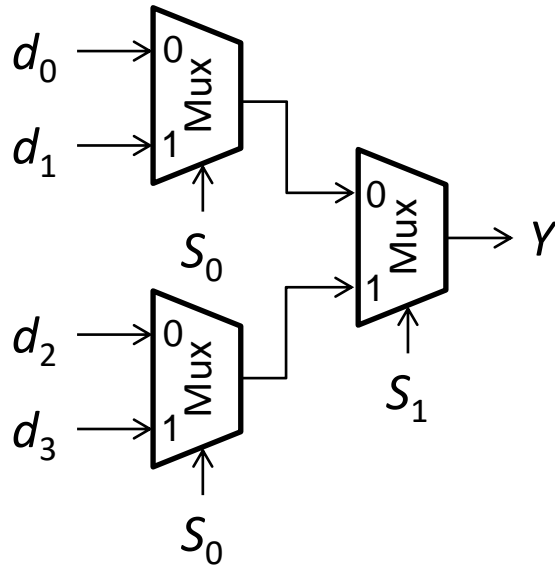
A Multiplexer can also be implemented using:

1. A decoder
2. Three-state gates

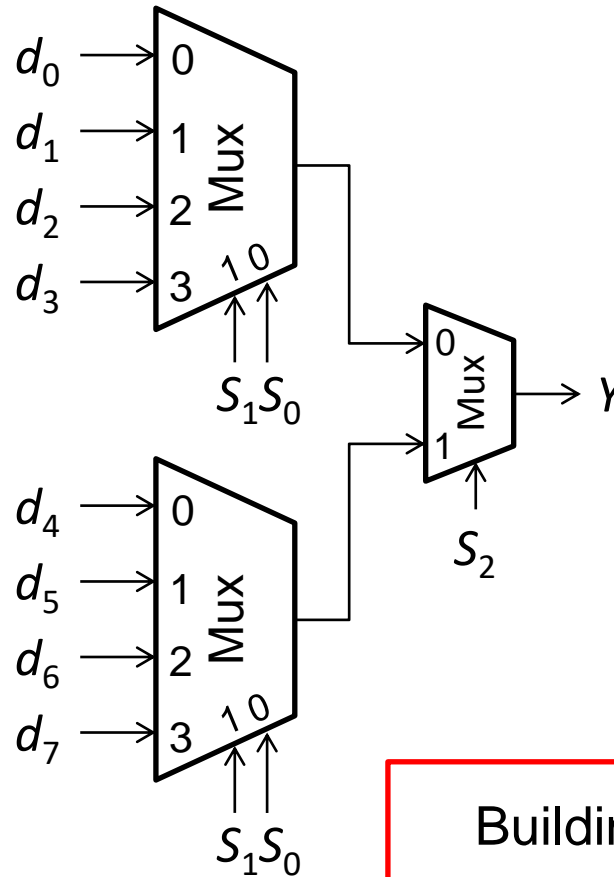


# Building Larger Multiplexers

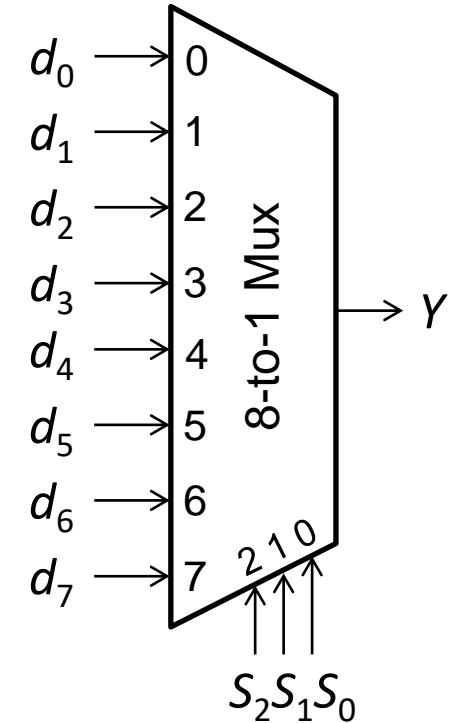
Larger multiplexers can be built hierarchically using smaller ones



Building 4-to-1  
Mux using three  
2-to-1 Muxes



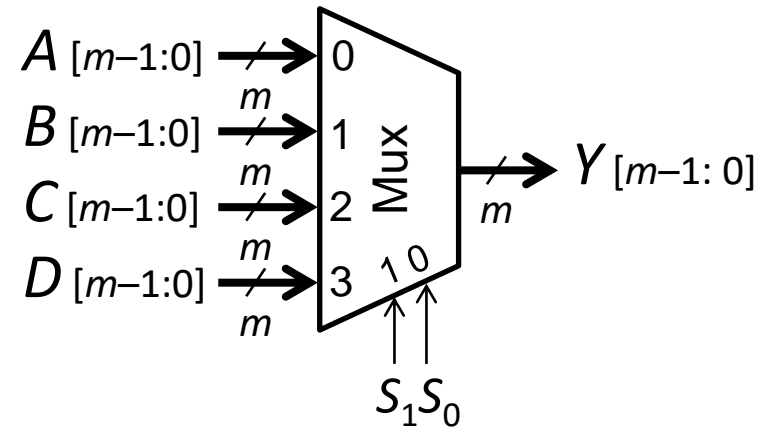
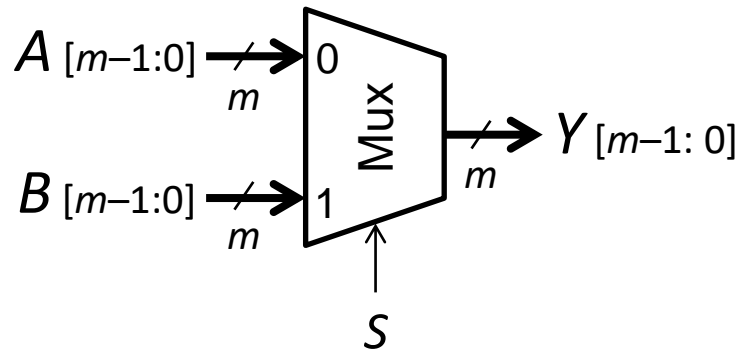
Building 8-to-1 Mux  
using two 4-to-1 Muxes  
and a 2-to-1 Mux





# Multiplexers with Vector Input and Output

The inputs and output of a multiplexer can be  $m$ -bit vectors



2-to-1 Multiplexer with  $m$  bits  
Inputs and output are  $m$ -bit vectors  
Using  $m$  copies of a 2-to-1 Mux

4-to-1 Multiplexer with  $m$  bits  
Inputs and output are  $m$ -bit vectors  
Using  $m$  copies of a 4-to-1 Mux

# Implementing a Function with a Multiplexer

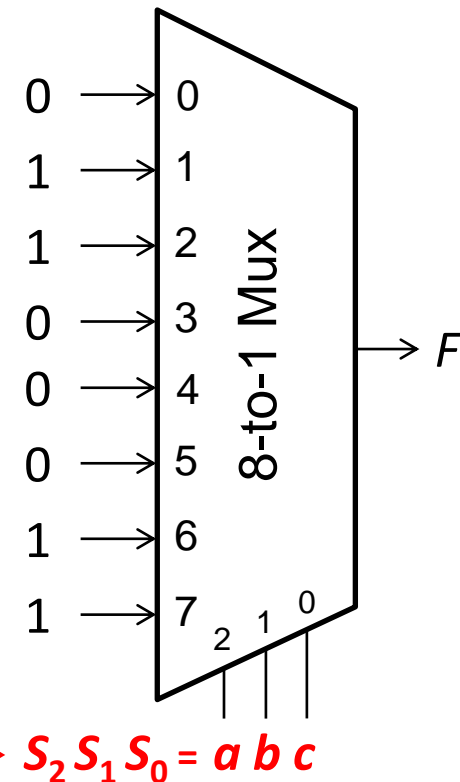
- ❖ A Multiplexer can be used to implement any logic function
- ❖ The function must be expressed using its minterms
- ❖ Example: Implement  $F(a, b, c) = \Sigma(1, 2, 6, 7)$  using a Mux

## ❖ Solution:

The inputs are used as select lines to a Mux.

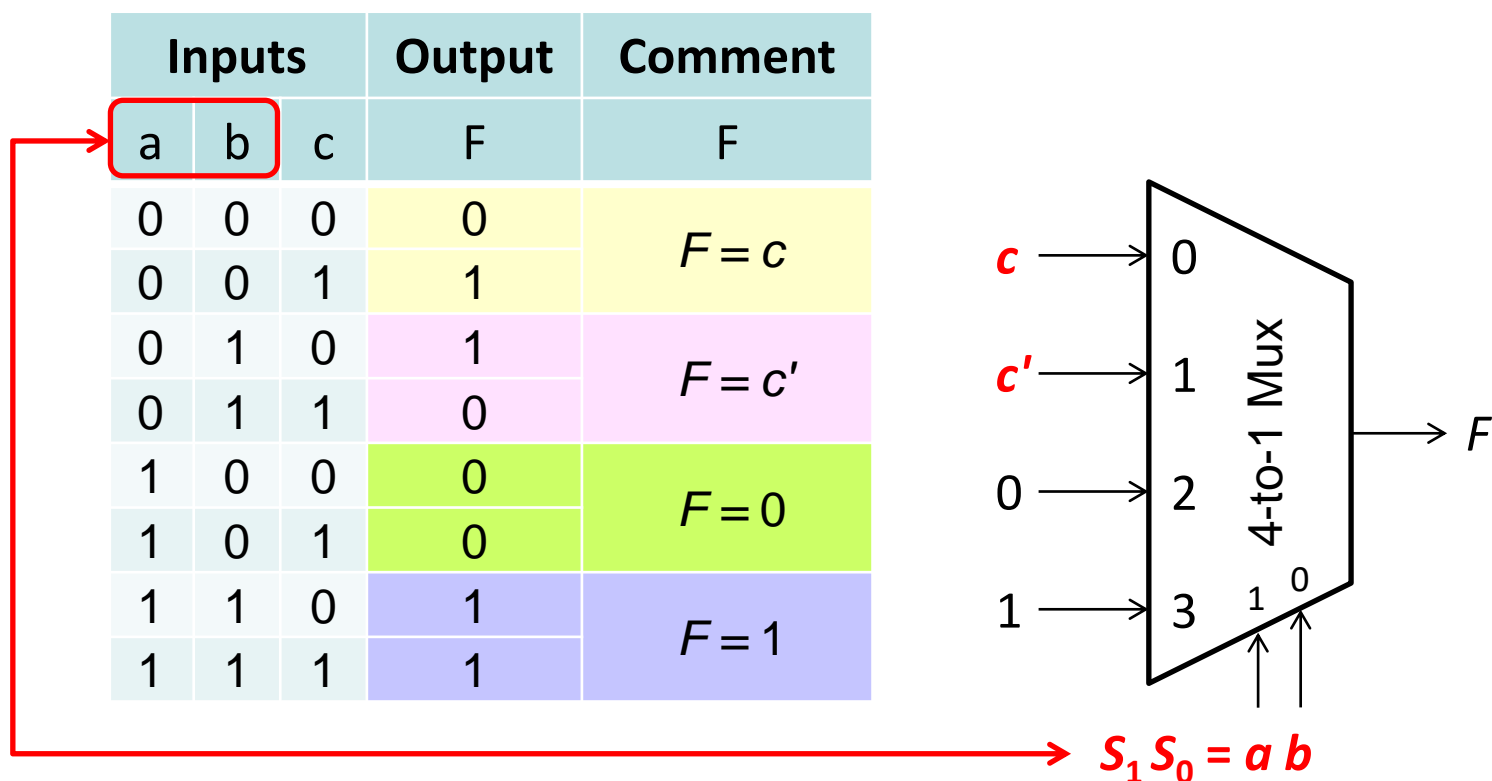
An 8-to-1 Mux is used because there are 3 variables

Inputs			Output
a	b	c	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1



# Better Solution with a Smaller Multiplexer

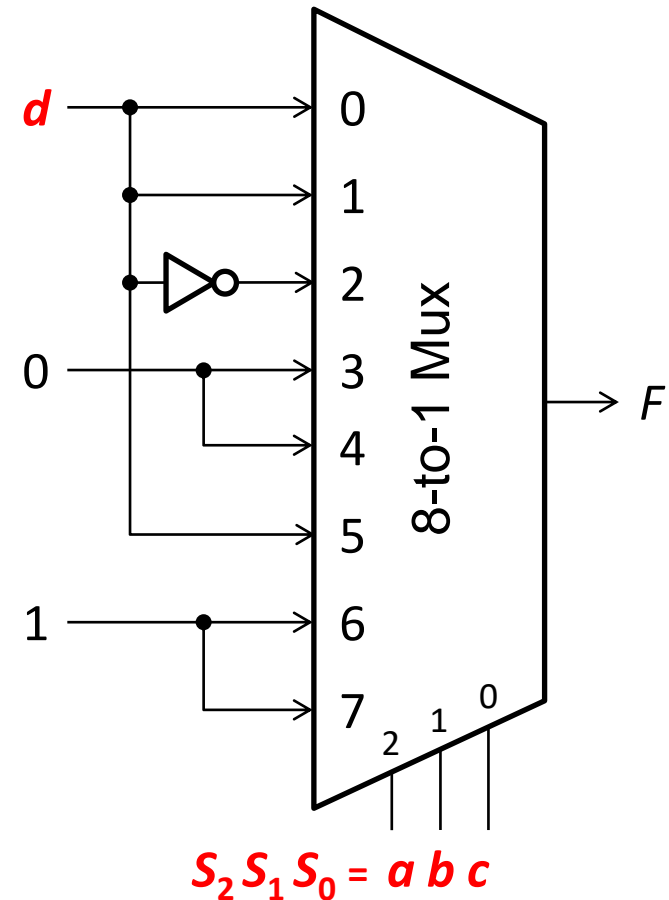
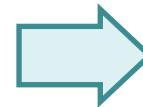
- ❖ Re-implement  $F(a, b, c) = \sum(1, 2, 6, 7)$  using a 4-to-1 Mux
- ❖ We will use the two select lines for variables  $a$  and  $b$
- ❖ Variable  $c$  and its complement are used as inputs to the Mux



# Implementing Functions: Example 2

Implement  $F(a, b, c, d) = \Sigma(1,3,4,11,12,13,14,15)$  using 8-to-1 Mux

Inputs				Output	Comment
<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>F</i>	<i>F</i>
0	0	0	0	0	$F = d$
0	0	0	1	1	
0	0	1	0	0	$F = d$
0	0	1	1	1	
0	1	0	0	1	$F = d'$
0	1	0	1	0	
0	1	1	0	0	$F = 0$
0	1	1	1	0	
1	0	0	0	0	$F = 0$
1	0	0	1	0	
1	0	1	0	0	$F = d$
1	0	1	1	1	
1	1	0	0	1	$F = 1$
1	1	0	1	1	
1	1	1	0	1	$F = 1$
1	1	1	1	1	



# Shannon's Expansion

- ❖ Shannon's Expansion Theorem:

$$f(x_1, \dots, x_i, \dots, x_n) = x_i' \cdot f_{x_i'} + x_i \cdot f_{x_i}$$

$$f_{x_i'} = f(x_1, \dots, x_i = 0, \dots, x_n) \text{ and } f_{x_i} = f(x_1, \dots, x_i = 1, \dots, x_n)$$

- ❖ Example:  $f(a, b, c) = ab + ac + bc$

$$f_{a'} = f(0, b, c) = 0 + 0 + bc = bc$$

$$f_a = f(1, b, c) = b + c + bc = b + c \quad (\text{absorption})$$

$$\text{Therefore, } f(a, b, c) = a'(bc) + a(b + c)$$

- ❖ Shannon's Expansion can be applied recursively:

$$f(a, b, c) = a'(b'(0) + b(c)) + a(b'(c) + b(1))$$

$$f(a, b, c) = a'b'(0) + a'b(c) + ab'(c) + ab(1)$$

# Shannon's Expansion (cont'd)

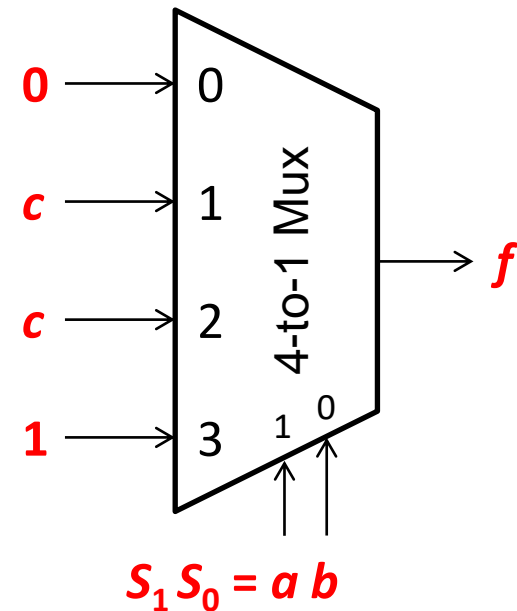
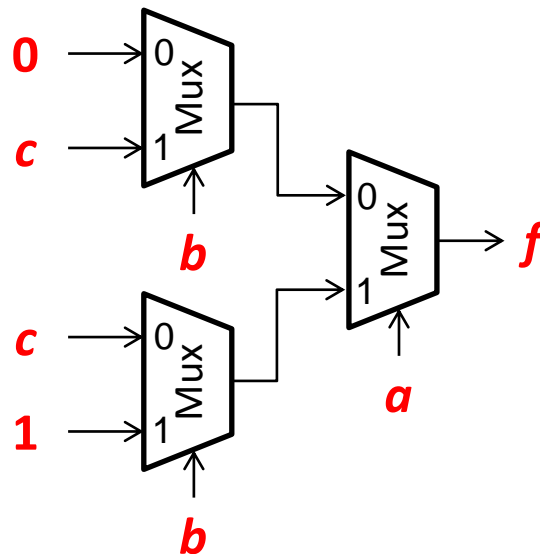
- ❖ Used to implement functions with multiplexers
- ❖ Example:  $f(a, b, c) = ab + ac + bc = a'(bc) + a(b + c)$

$$f(a, b, c) = a'(b'(0) + b(c)) + a(b'(c) + b(1))$$

$$f(a, b, c) = a'b'(0) + a'b(c) + ab'(c) + ab(1)$$

4×1 mux

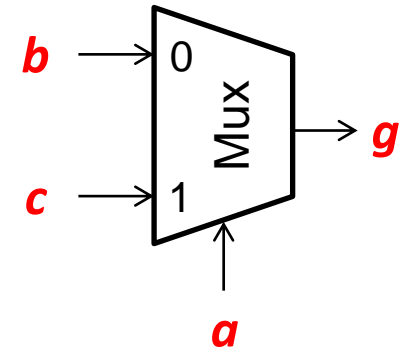
Three  
2×1 muxes



# Shannon's Expansion (cont'd)

❖  $g(a, b, c) = a'b + a c$

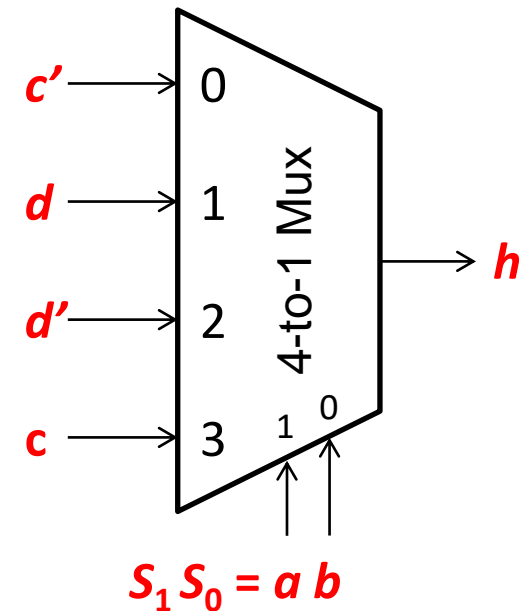
Can be implemented using a 2×1 mux



The truth table method uses a 4×1 mux

❖  $h(a, b, c, d) = a'b'c' + a'bd + ab'd' + abc$

Can be implemented using a 4×1 mux

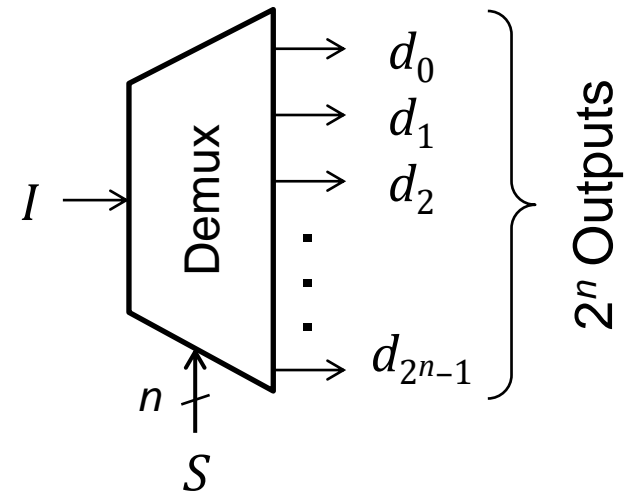


The truth table method uses an 8×1 mux

# Demultiplexer

- ❖ Performs the inverse operation of a Multiplexer
- ❖ A Demultiplexer (or Demux) is a combinational circuit that has:

1. One data input  $I$
2. An  $n$ -bit select input  $S$
3. A maximum of  $2^n$  data outputs



- ❖ The Demux directs the data input to one of the outputs

According to the select input  $S$



# Examples of Demultiplexers

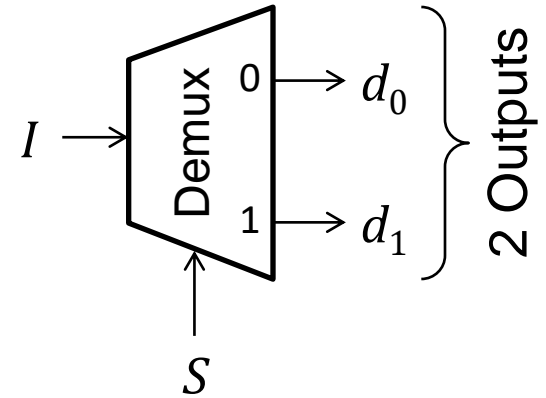
## ❖ 1-to-2 Demultiplexer

**if** ( $S == 0$ ) {  $d_0 = I$ ;  $d_1 = 0$ ; }

**else** {  $d_1 = I$ ;  $d_0 = 0$ ; }

Output expressions:

$$d_0 = I S'; d_1 = I S$$



## ❖ 1-to-4 Demultiplexer

**if** ( $S_1 S_0 == 00$ ) {  $d_0 = I$ ;  $d_1 = d_2 = d_3 = 0$ ; }

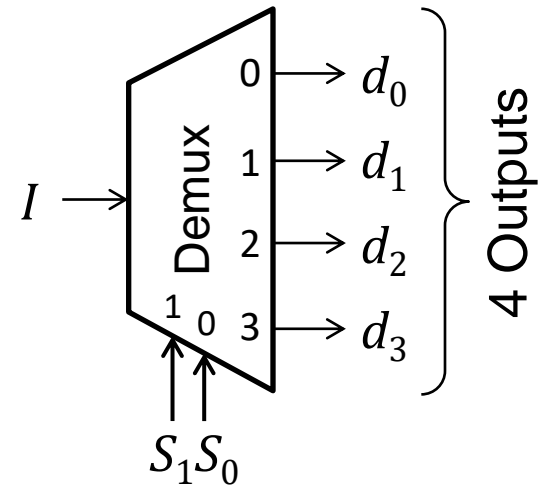
**else if** ( $S_1 S_0 == 01$ ) {  $d_1 = I$ ;  $d_0 = d_2 = d_3 = 0$ ; }

**else if** ( $S_1 S_0 == 10$ ) {  $d_2 = I$ ;  $d_0 = d_1 = d_3 = 0$ ; }

**else** {  $d_3 = I$ ;  $d_0 = d_1 = d_2 = 0$ ; }

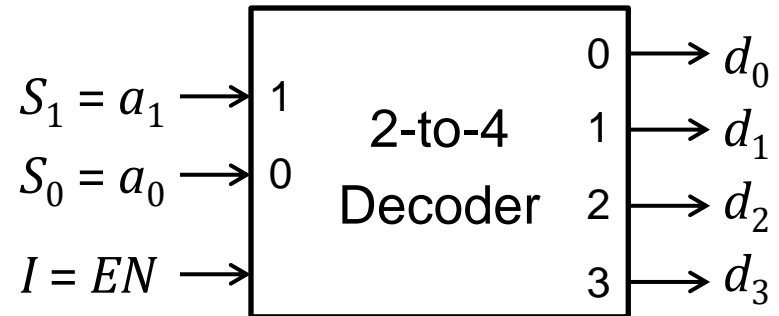
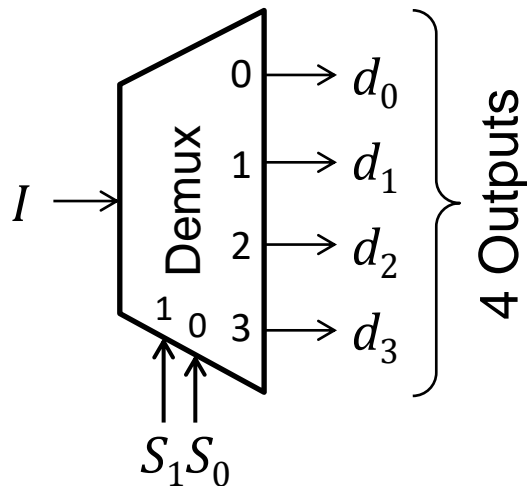
Output expressions:

$$d_0 = I S_1' S_0'; d_1 = I S_1' S_0; d_2 = I S_1 S_0'; d_3 = I S_1 S_0$$



# Demultiplexer = Decoder with Enable

- ❖ A 1-to-4 demux is equivalent to a 2-to-4 decoder with enable
- Demux select input  $S_1$  is equivalent to Decoder input  $a_1$
- Demux select input  $S_0$  is equivalent to Decoder input  $a_0$
- Demux Input  $I$  is equivalent to Decoder Enable  $EN$



Think of a decoder as directing the Enable signal to one output

- ❖ In general, a demux with  $n$  select inputs and  $2^n$  outputs is equivalent to a  $n$ -to- $2^n$  decoder with enable input

# Next . . .

- ❖ Decoders
- ❖ Encoders
- ❖ Multiplexers
- ❖ Demultiplexers
- ❖ Design Examples

# 2-by-2 Crossbar Switch

❖ A 2×2 crossbar switch is a combinational circuit that has:

Two  $m$ -bit Inputs:  $A$  and  $B$

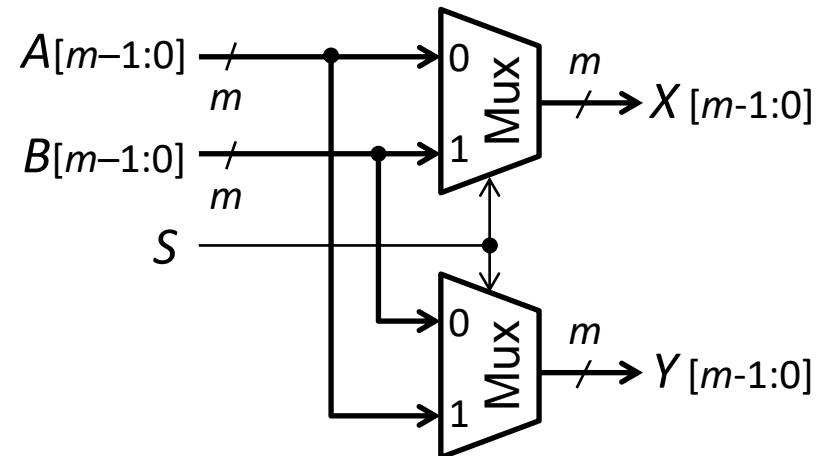
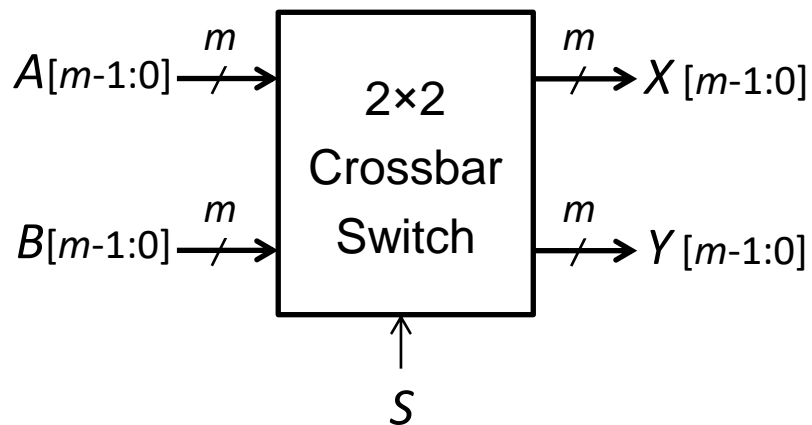
Two  $m$ -bit outputs:  $X$  and  $Y$

1-bit select input  $S$

```
if (S == 0) { X = A; Y = B; }  
else { X = B; Y = A; }
```

❖ Implement the 2×2 crossbar switch using multiplexers

❖ **Solution:** Two 2-input multiplexers are used



# Sorting Two Unsigned Integers

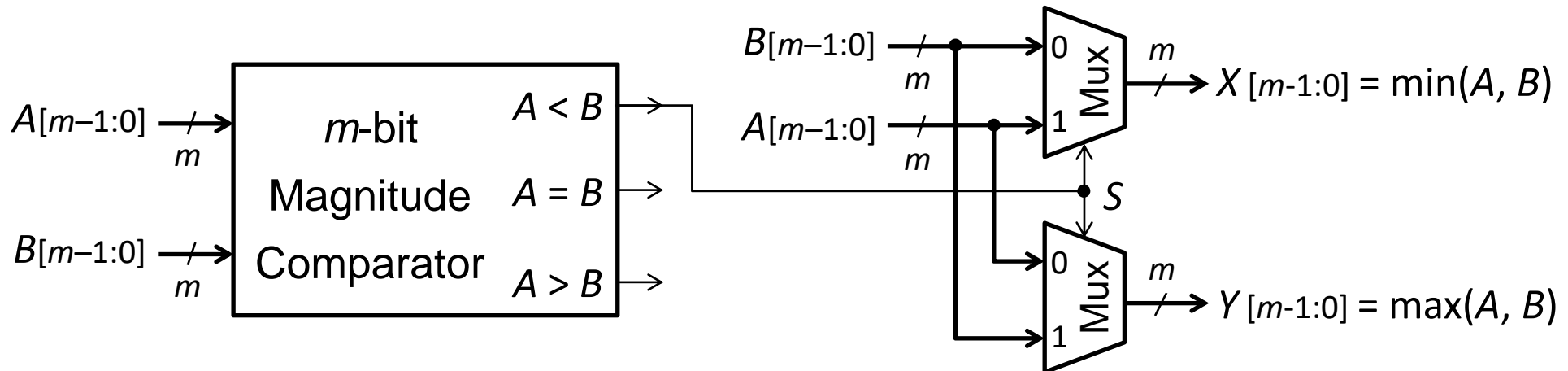
- ❖ Design a circuit that sorts two  $m$ -bit unsigned integers  $A$  and  $B$

Inputs: Two  $m$ -bit unsigned integers  $A$  and  $B$

Outputs:  $X = \min(A, B)$  and  $Y = \max(A, B)$

- ❖ **Solution:**

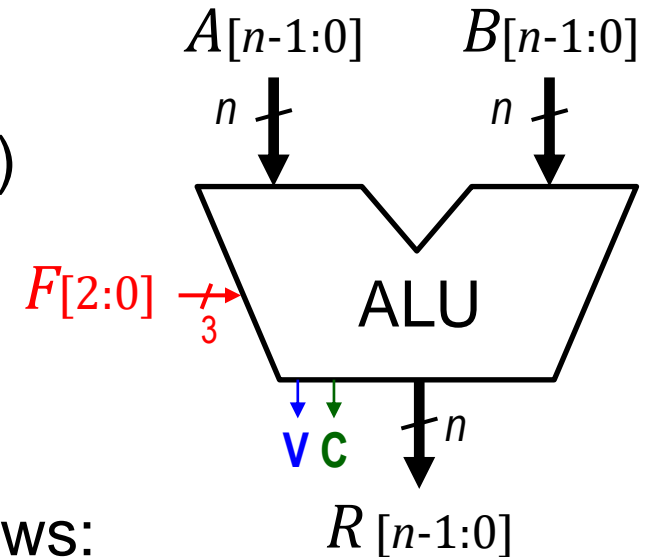
We will use a magnitude comparator to compare  $A$  with  $B$ , and  $2 \times 2$  crossbar switch implemented using two 2-input multiplexers



# Arithmetic and Logic Unit (ALU)

- ❖ Can perform many functions
- ❖ Most common ALU functions
  - Arithmetic functions: ADD, SUB (Subtract)
  - Logic functions: AND, OR, XOR, etc.
- ❖ We will design an ALU with 8 functions
- ❖ The function  $F$  is coded with 3 bits as follows:

## ALU Symbol



Function	ALU Result	Function	ALU Result
$F = 000$ (ADD)	$R = A + B$	$F = 100$ (AND)	$R = A \& B$
$F = 001$ (ADD + 1)	$R = A + B + 1$	$F = 101$ (OR)	$R = A   B$
$F = 010$ (SUB - 1)	$R = A - B - 1$	$F = 110$ (NOR)	$R = \sim(A   B)$
$F = 011$ (SUB)	$R = A - B$	$F = 111$ (XOR)	$R = (A \wedge B)$

# Designing a Simple ALU

